Free Vibration Analysis of Partially Buried Pipe in Elastic Foundation

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1. Introduction

In many industrial fields, cylindrical shells are widely used, because of their strength and effectiveness. For example, in petroleum industry, cylindrical shells are usually laid on the seabed as a foundation. Although they have important role, cylindrical shells are become critical for some safety issues, especially for their dynamic behavior. Therefore understanding free vibration behavior of cylindrical shells on the foundation will bring much useful information for analyzing and designing such structure. Yang [1] has investigated the whole buried pipelines under seismic loading using cylindrical shell element to model the pipelines. Purpose of this study is to introduce a method to analyze cylindrical shells partially buried in elastic foundation by means of finite strip method which is applicable for any end conditions.

2. Model and Formulation

Pipes are modeled by using thin isotropic elastic cylindrical shell element, assumed to be free from local instability of the shells. Soil as a foundation is modeled by elastic spring which is connected to shell in radial direction, but it is possible to include the axial, circumferential, and even the radial slope spring in this method as soon as the spring constants are known. Generalized model, strip and reference direction are shown in fig.1.

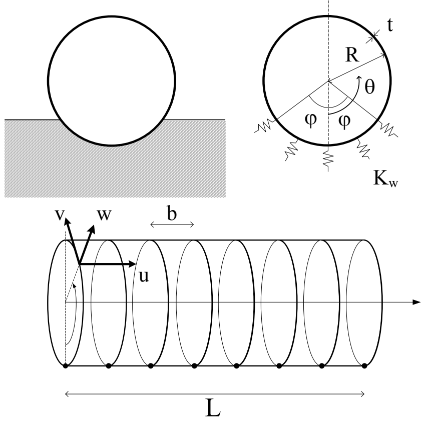

Fig. 1. Generalized model

$$\begin{aligned} u(x,\theta) &= \sum_{m=0}^{M} f_m^u(x)\cos(m\theta) = \sum_{m=0}^{M} \left[\mathbf{N}^u\right]_m \left\{\boldsymbol{\delta}_e^u\right\}_m \\ v(x,\theta) &= \sum_{m=0}^{M} f_m^v(x)\sin(m\theta) = \sum_{m=0}^{M} \left[\mathbf{N}^v\right]_m \left\{\boldsymbol{\delta}_e^v\right\}_m \\ w(x,\theta) &= \sum_{m=0}^{M} f_m^w(x)\cos(m\theta) = \sum_{m=0}^{M} \left[\mathbf{N}^w\right]_m \left\{\boldsymbol{\delta}_e^w\right\}_m \\ \beta(x,\theta) &= \sum_{m=0}^{M} f_m^\beta(x)\cos(m\theta) = \sum_{m=0}^{M} \left[\mathbf{N}^\beta\right]_m \left\{\boldsymbol{\delta}_e^\beta\right\}_m \end{aligned} \tag{1}$$

Using finite strip method [2], the displacement functions are defined by simple polynomial in longitudinal direction and continuous differentiable function in circumferential direction as given in eq.1. for an axisymmetric problem. The strains and curvature changes of the elastic cylindrical shell [3] are given in eq.2.

$$\{\boldsymbol{\varepsilon}\} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_\theta \\ \varepsilon_{x\theta} \\ \chi_x \\ \chi_\theta \\ \chi_{x\theta} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u}{\partial x} \\ \dfrac{1}{R}\dfrac{\partial v}{\partial \theta} + \dfrac{w}{R} \\ \dfrac{1}{R}\dfrac{\partial u}{\partial \theta} + \dfrac{\partial v}{\partial x} \\ -\dfrac{\partial^2 w}{\partial x^2} \\ -\dfrac{1}{R^2}\dfrac{\partial^2 w}{\partial \theta^2} + \dfrac{1}{R^2}\dfrac{\partial v}{\partial \theta} \\ 2\left(-\dfrac{1}{R}\dfrac{\partial^2 w}{\partial x \partial \theta} + \dfrac{1}{R}\dfrac{\partial v}{\partial x}\right) \end{Bmatrix} = \sum_{m=0}^{M} \left[\mathbf{B}\right]_m \left\{\boldsymbol{\delta}_e\right\}_m = \left[\mathbf{B}\right]\left\{\boldsymbol{\delta}_e\right\} \tag{2}$$

Keywords: Free Vibration, Finite Strip Method, Cylindrical Shells, Foundation

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The stiffness and mass matrix of the shell can be derived directly by using well known finite element formula as given in eq.3 and eq.4 respectively, in which **[D]**, ρ and **[N]** are elasticity matrix, mass per unit volume of shell and shape function.

$$\left[\mathbf{K_S}\right]=\int_V\left[\mathbf{B}\right]^T\left[\mathbf{D}\right]\left[\mathbf{B}\right]\,\mathrm{d}V \qquad (3)$$

$$\left[\mathbf{M_S}\right]=\rho\int_V\left[\mathbf{N}\right]^T\left[\mathbf{N}\right]\,\mathrm{d}V \qquad (4)$$

Since the foundation is not distributed uniformly along the circumference, the distribution function is used in order to have a partial distribution as given in eq.5 and stiffness matrix of foundation can be obtained by usage of eq.6.

$$\kappa(\theta)=\frac{K_w}{\pi}\left\{\varphi+\sum_{\ell=1}^{\infty}\left[\frac{2\ \sin(\ell\theta)}{\ell}\cos(\ell\theta)\right]\right\} \quad \text{, in which } K_w \text{ is radial spring constant} \qquad (5)$$

$$\left[\mathbf{K_F}\right]=\int_A\left[\mathbf{N}\right]^T\left[\mathbf{k_f}\right]\left[\mathbf{N}\right]\mathrm{d}A \quad \text{, in which } \left[\mathbf{k_f}\right]=\begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&\kappa(\theta)&0\\0&0&0&0\end{bmatrix} \qquad (6)$$

The integration can be done numerically or by observing the nature of trigonometric integration over the periphery of the cylindrical shells, by performing such observation then ℓ term can be taken infinite. The formation of foundation stiffness matrix implies the coupling phenomenon of term *m* and *n*, which is different from the shell stiffness matrix. After combining stiffness matrix of shell and foundation, the problem will be reduced to standard Eigenvalue problem which can be solved for natural frequencies and modes of the corresponding vibrations.

3. Numerical Results

Numerical example is presented here for simply support cylindrical shell with given geometry and foundation parameter as shown in fig.2 and fig.3 for variation of first and second lowest natural frequencies respectively. The convergence of the method depends not only on total number of strip but also total number of harmonic term. Large thin cylindrical shells on very stiff foundation need more total number of harmonic term to converge rather than small thick shells on soft foundation.

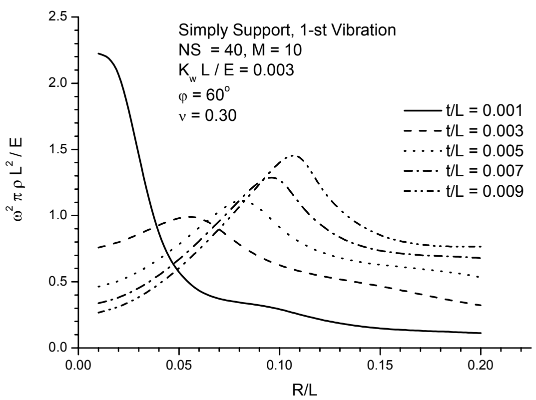

Fig. 2. Variation of first lowest natural frequency

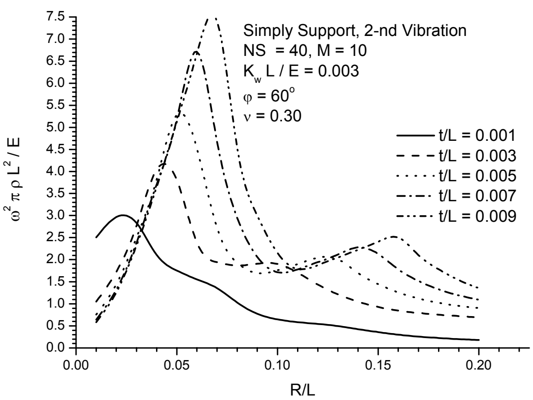

Fig. 3. Variation of second lowest natural frequency

4. Conclusions

Expansions of Fourier series in describing the foundation distribution and observation to the coupled harmonic terms have been employed successfully to formulate the foundation stiffness matrix, as a result sectional and transversal vibration mode can be observed simultaneously. Convergence of the method depends not only on the total number of strips but also on the total number of harmonic terms with consideration to the geometrical and foundation properties as well. More total number of harmonic terms is needed for higher vibrations.

References:

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