THERMO-MECHANICAL CONSTITUTIVE MODEL FOR HIGH DAMPING RUBBER

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1.0 Introduction

Rubber has many useful applications in engineering because of its special properties. Nowadays, many important products that meet high requirements are made of high damping rubber (HDR), such as vibration mounts, bearings for bridges and for seismic isolations. The HDR in bearings can be subjected to high frequency dynamic excitations, especially during earthquakes and dissipated considerable amount of heat increasing its body temperature. In previous work, authors have presented experimental investigations on temperature dependency behavior of HDR and its effects on mechanical properties[1]. This paper proposes a thermo-mechanical constitutive model for HDR. The model combined elasto-plastic body with strain dependent isotropic hardening law and hyperelastic body with damage model. Temperature dependent parameters, which can express rate dependent hardening, yielding and thermal softening are introduced to the model. Finally, the energy balance equation is constructed to evaluate average surface temperature of the material.

2.0 Modeling

A constitutive model is proposed, which combined hyperelasticity in parallel with elasto-plasticity. The viscosity is replaced by the equivalent plasticity. Compressibility of material has assumed and modeled in hyperelastic part. The rate dependent behaviors of material are modeled using the temperature. The rate dependent hardening and thermal softening are modeled in hyperelastic part. The complete formulation for hyperelastic part given as follows

$$\bar{W} = gW_1 + hW_2 + W_3 \tag{1}$$

$$W_1 = c_1(\bar{I}_c - 3) + c_2(\bar{II}_c - 3)\,,\; W_2 = \frac{c_3 c}{n+1}\left(\frac{\bar{I}_c - 3}{c}\right)^{n+1}\,,\; W_3 = a_\theta \frac{\theta}{\theta_0}(\bar{I}_c - 3)^{b_\theta} - c_\theta \theta (\bar{I}_C - 3)^{d_\theta} \tag{2a,b.c}$$

$$g(x) = \beta + (1-\beta)\left[\frac{1-e^{-x/\alpha}}{x/\alpha}\right],\; x(t) = \max_{s\in(-\infty,t]} \sqrt{2W_1(s)} \tag{3a,b}$$

$$h(y) = 1 - \frac{1}{1+\exp\{-a_H(y-b_H)\}},\; y(t) = \max_{s\in(-\infty,t]}\left(\bar{I}_c(s) - 3\right) \tag{4a,b}$$

where $\bar{I}_c, \bar{II}_c$ are reduced first and second invariant of right Cauchy green tensor respectively; c_1, c_2, c_3, c, n α, β, a_H and b_H are material constants while $a_\theta, b_\theta, c_\theta, d_\theta$ are temperature dependent material constants and θ, θ_0 are average maximum surface temperature of material and room temperature respectively.

Elasto-plastic constitutive law for energy absorbing material has been proposed with specific properties such as unloading stiffness, hardening[2]. The law is described by a differential equation (Eq.5a) and judgment for yielding and unloading is conducted automatically.

$$\dot{\mathbf{T}}_{(J)} = \mathbf{C}^{(E)} : (\mathbf{D} - \mathbf{D}^P)\,,\; \mathbf{D}^P = (3k_2)^{1/2}(3J_2)^{(N-1)/2}\frac{\mathbf{T}'}{t_y}\;,\; k_2 = \frac{\mathbf{D}':\mathbf{D}'}{2}\,,\; J_2 = \frac{\mathbf{T}':\mathbf{T}'}{2t_y^2} \tag{5a,b,c,d}$$

where τ_y, N are yield stress and constant respectively. From the experimental results, it is observed that yield stress is not only function of invariants but also a function of temperature (velocity). Therefore, following function is proposed for yield stress that represents both hardening and temperature effect.

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$$\tau_y = \tau'(\theta,\theta_0)\left\{1+\left(\frac{\bar{I}_c-3}{c}\right)^b\right\},\quad \tau' = (\tau_0/e_\theta)\frac{\theta}{\theta_0} \qquad \textbf{(6a,b)}$$

where τ_0, b, e_θ are initial yield stress and two material constants respectively. In this model, an elastic constitutive tensor $C^{(E)}$ is defined based on hyperelasticity with strain energy density function W_E while strain function includes temperature dependency function. Complete derivation of elastic tensor with W_E is shown as follows

$$C^{(E)}_{pqrs} = \frac{1}{J}F_{pi}F_{qj}F_{rk}F_{sl}C^{(0)}_{ijkl} + \delta_{pr}T^{(h)}_{sp} + \delta_{qs}T^{(h)}_{pr} - \delta_{rs}T^{(h)}_{pq},\ C^{(0)} = \frac{\partial^2 W_E}{\partial\mathbf{E}\partial\mathbf{E}},\ \mathbf{T}^{(h)} = \frac{1}{J}\mathbf{F}.\frac{\partial W_E}{\partial\mathbf{E}}.\mathbf{F}^T \qquad \textbf{(7a,b)}$$

$$W_E = c_4(\bar{I}_C-3)+c_5(\bar{II}_C-3)+\frac{c_4 c}{m+1}\left(\frac{\bar{I}_C-3}{c}\right)^{m+1} + f_\theta\frac{\theta}{\theta_0}(\bar{I}_C-3) \qquad \textbf{(8)}$$

where c_4, c_5, m and f_θ are material constants. Fig. 1 shows the comparison of the experimental results with a model when loading frequency is equal to 1Hz. It shows the good agreement except virgin loading.

3.0 Temperature evaluation

In order to evaluate the average temperature using the model, thermal balance equations is constructed. In this problem, we assumed convective heat loss at the boundaries and no local conduction. Internal production of energy results from irreversible work is simply calculated by time integration of stress-strain produced in the model. It is assumed that 95% of irreversible work is convert into the heat. The simplified energy balance equation is given as follows

$$\rho C\frac{d\theta}{dt} = Q_{gen} - \frac{hA}{V}(\theta-\theta_0),\ Q_{gen} = \frac{0.95}{T}\int_0^T \tau\, d\gamma \qquad \textbf{(9a,b)}$$

where C, h ,V, A, τ ,γ and T are specific heat capacity of material, convective heat transfer coefficient, volume, convective area, shear stress, shear strain and time respectively. The solution of differential equation in Equation (9a) gives average surface temperature. Fig. 2 shows temperature evaluation by the model and comparison with the experimental results. It shows temperature evaluation model can be used to predict average surface temperature.

4.0 Conclusions

A constitutive model is proposed for HDR. The model consists of energy absorbing properties, hardening properties, viscosity properties and thermal effects. The model shows good agreement with experimental results. A present study can be used as a key for thermo-mechanical finite element modeling for high damping rubber bearings.

References

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2. Yoshida J., Abe M., Fujino Y., "Constitutive law for high damping rubber materials," *J. Engineering Mechanics (ASCE)*, (submitted).

Figures

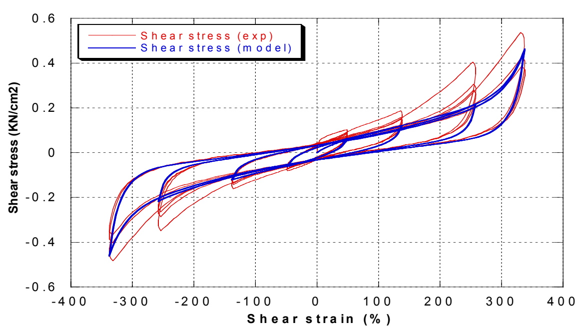

Fig. 1: Stress-strain relation (f=1 Hz)

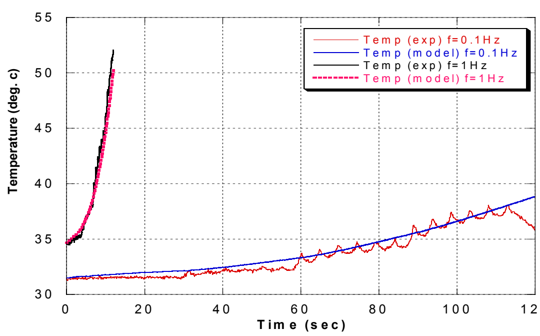

Fig. 2: Temperature comparison