Ground Motion Simulation in Train Track/Ground Dynamics

Student member of JSCE Bian Xuecheng, Ph.D Candidate, Okayama University
Member of JSCE Hirokazu Takemiya, Professor, Okayama University

Background

Vibration of an integrated system comprising the railway track and ground under the action of a moving train is of great practical importance to civil and transportation engineering. Analysis of the waves that are generated in the structure and ground by trains is necessary for a proper assessment of deterioration and safety of the railway system. Train track dynamics have been dealt with practically by a beam model on Winkler foundation. Recently the ground condition has been sophisticated either by a half space or a layered system or a stack of layers for the wave propagation in subsoil media[1]. The wave propagation mechanisms are different in varied ground conditions. In case of a half space assumption, the wave field is predominantly governed by the Raleigh wave. In case of a layer/layers assumption, on the other hand, the dispersive nature appears and the wave field is governed by the modal waves which can be characterized by different wave speeds for different frequencies. The energy transmission is carried out by the modes that are most concerned with the situation. These facts demand the more careful consideration of ground geometry.

The sleepers in the track system work to transfer the load from rail to the ground. In former works, the track is modeled as a beam resting on ground, thus the load is acting on ground as a spatial variable. But with the presence of sleepers, the load pattern on the ground will be changed to a spatial stationary, but time-dependent variable. Recently, the existence of sleepers and the spatially distribution of the sleeper supports are also recognized but in the continuous treatment[3]. But for the ground motion under discrete sleeper's loads, there are little references yet. Furthermore, the more complicit model with layered soil ground needs to be introduced to cooperate with the superstructures. J.J. Kalker's discrete supported rail model[2] has been extended to layered soil herein.

Track/ground interaction under train loads

First, a multi-supported Euler beam motion equation had been studied in frequency domain, and then the layered ground is introduced to cooperate with the superstructure's vibration via sleepers. **Fig.1** depicts the model.

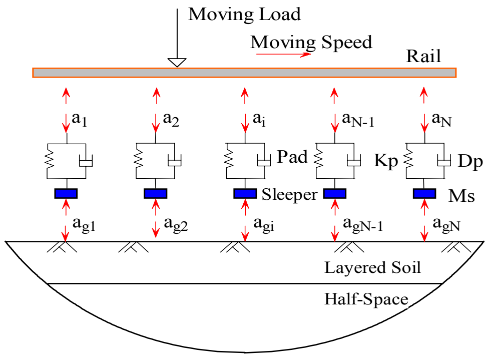

Fig.1 Track/ground model

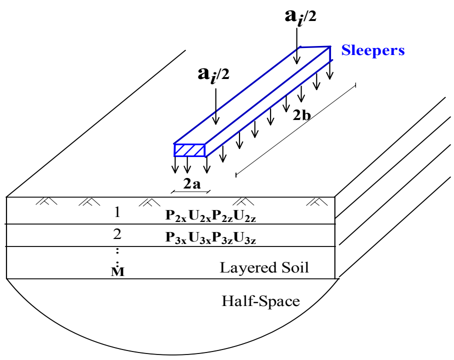

Fig. 2 Sleeper-ground model

Flexibility matrix of ground under multi-sleepers' loads

Accounting for the sleeper's geometry and size and under a concentrated load a_i, a uniform stress distribution on ground surface over rectangle shape is assumed here. The sleeper's size is $2a \times 2b$ (**Fig. 2**). Hence, the stress intensity under the sleeper is $a_i / 4ab$. The ground responses u^{txy}, v^{txy}, w^{txy} are then derived in close form [3]. For the case of an array of ***N*** sleepers acting on ground surface, the load in the wavenumber domain can be expressed as $P^{xy} = \sum_{i=1}^{N} a_{gi}^{t} e^{-\beta x_i}$. Therefore the vertical ground responses in space domain via inverse Fourier transform lead to:

$$w(x,y,\omega) = \sum_{i=1}^{N} a_{gi}^{t} \int_{-\infty}^{+\infty} -Q_{33}(\beta,\gamma) \frac{\sin \beta a}{\beta a} \frac{\sin \gamma b}{\gamma b} e^{i\beta(x-x_i)} e^{i\gamma y} d\beta d\gamma \tag{1}$$

The relation of vertical response on ground surface at sleeper's positions $\mathbf{U}_g$ and the loads transferred from sleepers $\mathbf{A}_g$ can be expressed as,

$$\mathbf{U} = \mathbf{H}_g \mathbf{A} \tag{2}$$

elements of $\mathbf{H}_g$ can be expressed as,

$$\mathbf{H}_{g,ij} = \int_{-\infty}^{+\infty} -Q_{33}(\beta,\gamma) \frac{\sin \beta a}{\beta a} \frac{\sin \gamma b}{\gamma b} e^{i\beta(x_j - x_i)} d\beta d\gamma\ , \quad j,i = 1,2,\dots N \tag{3}$$

KEY WORDS: Surface Wave, Layered soil, Ground response, Dynamic flexibility matrix, Sleeper effect
〒700-8530 岡山市津島中 3-1-1 岡山大学環境理工学部環境デザイン工学科 Tel(Fax) 086-2518146

Sleeper's effect

The railpad are described by a stiffness spring K_p and a damping coefficient D_p, and a sleeper mass is M_s. From the force equilibrium and Newton's second theorem, the flexibility matrix for the deflections of rail at sleepers' positions due to the support forces from sleepers is obtained as,

$$\mathbf{H} = -[(\mathbf{I} - M_s\omega^2\mathbf{H}_g)^{-1}\mathbf{H}_g + 1/(K_p + i\omega D_p)\mathbf{I}] \tag{4}$$

where **I** is a unit matrix of $N \times N$ size. When the interaction forces **A** between rail and sleepers are solved, the forces on the ground $\mathbf{A}_g$ is obtained as,

$$\mathbf{A}_g = -(\mathbf{I} + M_s\omega^2\mathbf{H}_g)^{-1}\mathbf{A} \tag{5}$$

Computation and results

According to formulation deduced above, the ground displacement at position(x, y, z=0) on ground surface have been computed with the train parameter and ground soil profile list below, and the train wheel axels loads configuration of Sweden X2000 are used. Results of rail vibration and ground motion are shown in **Fig.3-Fig.6.**

Soil profile in numerical implementation

Soil Layer	Thickness (m)	Mass Density (kg/m^3)	Shear Velocity V_S (m/s)		Poison Ratio	Damping ratio	
			C=70km/h	C=200km/h		C=70km/h	C=200km/h
Embankment	1.4	1,800	250.0	150.0	0.49	0.04	0.04
Surface Crust	1.1	1,500	72	65	0.49	0.04	0.063
Organic clay	3.0	1,260	41	33	0.49	0.02	0.058
Clay	4.5	1,475	65	60	0.49	0.05	0.098
Clay	6.0	1,475	87	85	0.49	0.05	0.064
Half-space	-	1,475	100	100	0.49	0.05	0.060

Rail parameter in numerical implementation

Mass of rail per meter (kg)	Flexural rigidity of rail (MN.m^2)	Rail-pad Spring const (MN/m)	Rail-pad Damping const.(KN.s/m)	Mass of sleeper (kg)
56.0	4.86	110.0	650.0	205

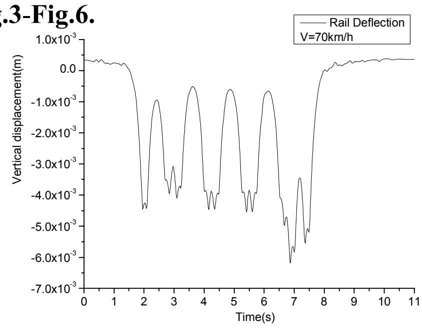

Fig. 3 Rail disp. under train loads (V=70km/s)

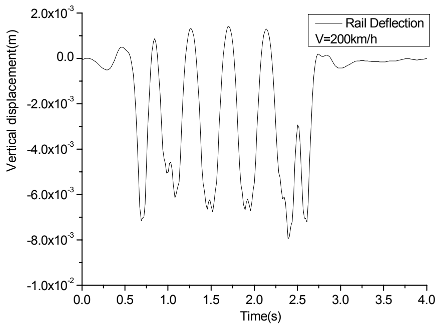

Fig. 4 Rail disp. under tain loads (V=200km/s)

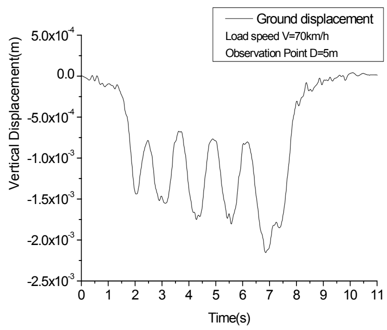

Fig. 5 Ground disp. under train load (5m from rail centreline,V=70km/h)

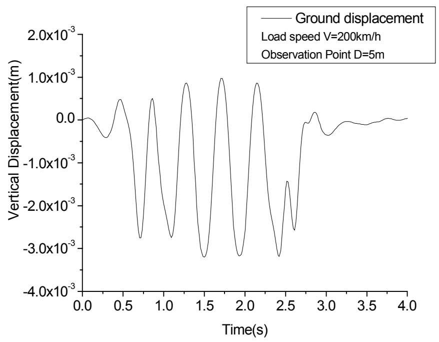

Fig. 6 Ground disp. under train load (5m from rail centreline,V=200km/h)

Conclusions

A substructure method has been developed here to solve the dynamics problem of a railway track and ground motion along rail line. The transfer matrix method is utilized here for the formulation of layered ground relying on half-space in order to derive ground flexibility matrix under the sleepers' loads. In contrast to the traditional continuous track ground contact assumption, the analytical results of layered ground can be used to study the dynamics phenomena from more reality aspects.

The rail and ground motions are computed for two different velocities. The motions of the track and ground have been found to increase with the increasing of train speed until a critical velocity is exceeded. The loads from sleepers onto the ground have been transformed from spatial variables to time-dependent variables which is advantageous to the ground motion simulation. The computation results show that the discrete properties of supports affect the behaviour of rail vibration more in low frequency range than in high frequency range. And also, the role of sleeper is more conspicuous for low speed train operation than for high speed case.

References:

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