Vibration of discretely supported rail on layered ground due to the moving load

Student member of JSCE Bian Xuecheng, Ph.D Candidate, Okayama University
Member of JSCE Hirokazu Takemiya, Professor, Okayama University

Introduction

The dynamic response of an Euler beam subjected to a moving load has been analyzed in a number of papers in order to study vibrations of a railroad. In most papers, the track subsoil (including sleeper, ballast and embankment) is replaced by springs of constant stiffness placed under beam.

In this paper, we will explore the behaviour of rail supported by discrete sleepers. As we know, while the rail vibrates, it generates elastic waves in the rail-pad-ballast system and surrounding subsoil. The reaction of these waves against the sleepers depends on the frequency of the waves. Therefore, sleeper-pad and subsoil will be considered as a frequency-dependent stiffness. J.J.Kalker (1996) studied the rail vibration behaviors in his works with dynamic stiffness of Pad-Sleepers and ballast system on rigid half-space[1]. Actually, this paper is an extension of his instructive works.

When the ground is modeled as Winkler foundation model with continuous springs support, the effect of sleepers, pads and ballast are generally included in the track stiffness. But it's difficult to give an accurate estimation of the stiffness of embankment because of the discrete sleepers supports. In this paper, the rail is studied as an Euler beam supported by discrete sleepers. This will be convenient to determine the dynamic stiffness of substructure, and also, enable us to watch which sleepers will be affected when the load passes by one specific sleeper.

The assumption to replace ground reaction with springs of Winkler foundation model also has an implicit assumption that the springs work independently, but as a matter of fact; the displacement of the location of one sleeper will inevitably cause the consequent movement of the neighboring sleepers. In this paper, the dynamic stiffness of layered soil method is adopted to study the dynamic response of soil to the multi-concentrated load on the surface to consider their interactive effects.

Modeling and Analysis

The rail is modeled as an Euler beam supported by sleepers at the positions of x_i with pads. The pad is replaced by a spring and a damper, and the reaction force to the rail at x_i is a_i. The moving single load has constant intensity P_0. Each sleeper has mass of Ms, and is supported by the ballast and embankment, which is modeled as a layer of ground. And the reaction from ground to sleeper is a_{gi} at x_i. The interactions between rail, pad, sleepers and ground are presented in **Fig1**. The displacement of rail can be described by the partial differential equation respecting vertical displacement of rail. And at the position of sleepers, the rail satisfies the compatible conditions of displacements with the sleepers and pads. Applied the Fourier transform to the equations with the coordination x and time t, the displacement of rail can be obtained in analytical form with Green function. To get the solution in frequency domain, the inverse Fourier transform with wave number will be performed on the Green function. And the analytical result can be obtained from the residue theorem and convolution theory. Also we can get the time history of rail vibration with another inverse Fourier transform with frequency, and it is convenient to take advantage of FFT algorithm. The reaction forces of sleepers are to be determined by the compatible condition of displacement with sleepers.

The sleepers and pads beneath the rail transfer the load from the rail to the ground. For the multi-loads act on sleepers, the transfer matrix can be got with the Newton's second theorem and the compatible conditions of displacements on the locations of sleepers. The stiffness matrix achieve by Haskell-Thompson via transfer matrix method and its elegant implementation by Eduardo Kausel, et. al.[2,3] will be utilized here to generate the dynamic stiffness of the layered ground under discrete and concentrated loads. The ballast and embankment are modelled as a layer with certain shear velocity. After integrating the stiffness matrix of ground with that of the rail in frequency domain, the reactions of sleepers to the rail a_i can be got by Gauss elimination method. Substituting a_i into the analytical solution of rail vibration, the displacement of rail in frequency domain will be got. Thereafter, the time history of rail vibration can be obtained with inverse FFT algorithm.

Soil profile in numerical implementation

Soil Layer	Thickness (m)	Mass Density (kg/m^3)	Shear Velocity V_S (m/s)		Poison Ratio	Damping ratio	
			C=70km/h	C=200km/h		C=70km/h	C=200km/h
Embankment	1.4	1,800	250.0	150.0	0.49	0.04	0.04
Surface Crust	1.1	1,500	72	65	0.49	0.04	0.063
Organic clay	3.0	1,260	41	33	0.49	0.02	0.058
Clay	4.5	1,475	65	60	0.49	0.05	0.098
Clay	6.0	1,475	87	85	0.49	0.05	0.064
Half-space	-	1,475	100	100	0.49	0.05	0.060

Rail parameter in numerical implementation

Mass of rail per meter (kg)	Flexural rigidity of rail (MN.m^2)	Rail-pad Spring const (MN/m)	Rail-pad Damping const.(KN.s/m)	Mass of sleeper (kg)
56.0	4.86	110.0	650.0	205

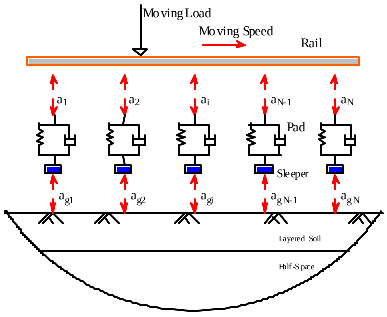

Fig1. Model of Rail-Sleeper-Ground

In numerical implementation, the rail was considered resting on 61 sleepers (Except the case of multi-wheel loads, there are 150 sleepers needed), the 30 sleepers lie to the left of the origin situated in sleeper 31. Motion takes place from left to right, and load starts at the origin point, at t=0. The sleepers are equally spaced with 0.7m apart. Two moving speed of load 70km/h and 200km/h was considered, which represent low speed and high speed respectively. In **Fig2** is a comparing of time history of

KEY WORDS: Layered soil, Discretely supported beam, Dynamic stiffness,
〒700-8530 岡山市津島中 3-1-1 岡山大学環境理工学部環境デザイン工学科 Tel(Fax) 086-2518146

displacement of 5th sleeper under the single load with 70km/h and 200km/h. The vibration of 2nd sleeper under multi-wheel loads with speed of 70km/h was also showed in **Fig3,** and **Fig4** for the speed of 200km/h. **Fig5** is the profile of multi-axels loads of train for computation.

In **Fig6**, the effect lengths of rail when load passes the right 2nd sleeper with speed of 70km/h and 200km/h are presented. The effect length increases with the moving speed grows. And from the numerical experiments, we find that the effect length is highly dependent on the stiffness of the rail.

The load transferred from the rail to the sleepers also will be available from the results we get, which can be used in the ground motion simulation. In **Fig7**, the reaction force on 8th sleeper when the load passes by is presented.

Conclusions

In this paper, the vertical displacement of rail supported by discrete sleepers has been studied, and the dynamic stiffness of layered ground is utilized which is more close to reality conditions. The analytical results are obtained in frequency domain, and the time histories of rail vibration are also available from inverse FFT both for single and multi-axels loads. When the load moves passing by the sleeper position, the length it will affect to the neighboring sleepers is achieved, this effect length is difficult to determine in the former efforts with the assumption of continuous spring-supports of Winkler foundation. The reaction to the rail from the ground is discrete. Although the computation results show that the discrete property of supports will not change the behavior of rail vibration much because of its large stiffness; it will give a new type load to simulate the ground motion caused by the moving train.

References:

1. J.J.Kalker, Discretely Supported Rails Subjected to Transient Loads. Vehicle System Dynamics, 25(1996), pp. 1-88
2. Eduardo Kausel and Jose Manuel Roesset, Stiffness Matrices for Layered Soils. Bulletin of the Seismological Society of America, Vol.71, No.6, pp.1743-1761, December 1981
3. Amir M. Kaynia, Christian Madshus, and Peter Zackrisson, Ground Vibration from High-Speed Trains: Prediction and Countermeasures. Journal of Geotechnical and Geoenvironmental Engineering, June 2000, pp. 531-537

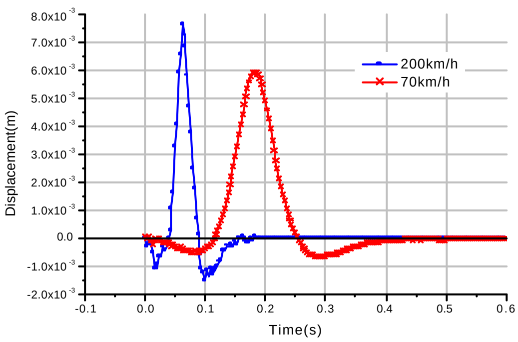

Fig2. Displacements of rail at 5th sleeper at different speeds

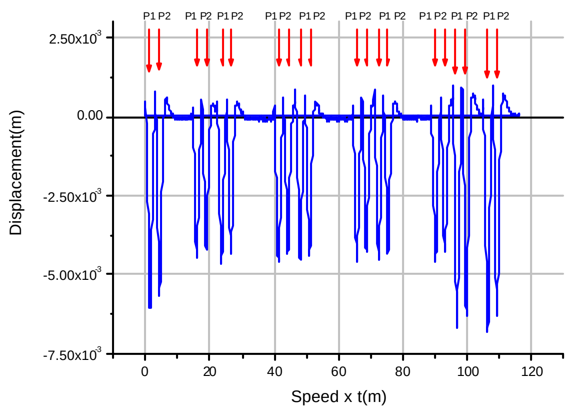

Fig3. Vibration of 2nd sleeper under multi-wheel axels loads at speed of 70km/h

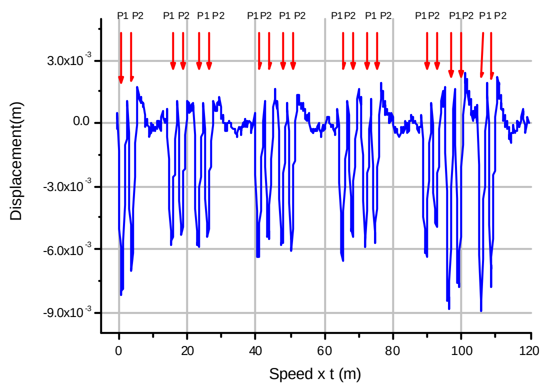

Fig4. Vibration of 2nd sleeper under multi-wheel axels loads at speed of 200km/h

Car NO. (Left to right)	P1 (KN)	P2 (KN)
1	160	117.5
2	122.5	122.5
3	122.5	122.5
4	122.5	122.5
5	180	181.5

Fig 5. Profile of multi-wheel axels Loads

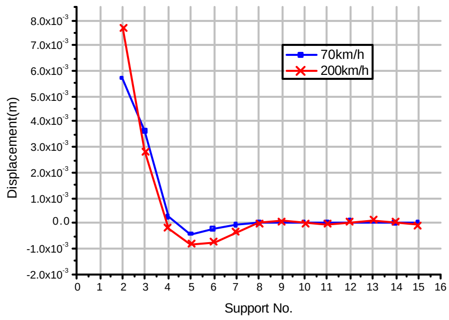

Fig6. Effect length of sleepers when load passes by 2nd sleepers

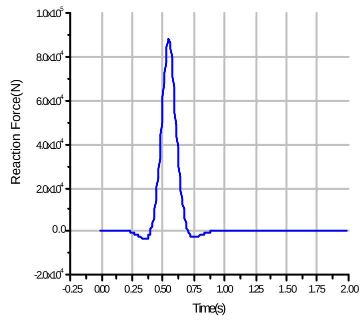

Fig7. The reaction force of 8th sleeper when load passes with speed of 70km/h