# **CAPACITY RELIABILITY OF NETWORK UNDER NODE CAPACITY CONSTRAINTS**

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## **1.** INTRODUCTION

The capacity of a network is usually defined as the maximum origin-destination demand that can be accommodated into the network without violating the specified capacity of each link, and its variation is reserve network capacity (Wong, 1996, 1997). The network, however, consists of elements including links nodes, which can be translated naturally into basic segments and intersections, respectively. In an urban street network, most of bottlenecks can be observed at intersections rather than the basic segments. There thus be necessary to study the network capacity and its reliability under node capacity constraints.

With increasing demand for better and more reliable services, the more attention has been concentrated on the reliability analysis of a road network. In the article, the network capacity reliability under node capacity constraints is investigated, and compared with those under link capacity constraints (Chen, 1999). To guarantee the quality of service provided by road transportation system, the node capacity conclusively should not be devaluated in evaluating performances of the road network.

## **2. NODE CAPACITY CONSTRAINTS**

Consider a signalized intersection with three or more approaching links. Let  $g_n^i$  be the green time given to link *i* approaching node n, L the total lost time of all phases per cycle, C the cycle length. Green intervals and lost time must satisfy the relationship

 $\sum_{i} g_{n}^{i} + L_{n} = C_{n}$  or  $\sum_{i} g_{n}^{i} / C_{n} = 1 - L_{n} / C_{n} \le 1$ 

The term in the left hand side of the last equation signifies the proportion of the available green time to a signal cycle, which can be characterized as the equalization of the composite of flow ratios for all the critical flow movements. Let  $A_n^i$  denote the set of links connecting to node n, during signal phase i. The access restriction of signaliezed intersection may be written as

 $\sum_{i} \max_{a} \left\{ x_{a} / c_{a} \middle| a \in A_{n}^{i} \right\} \le 1$ This node pass inequality signifies a kind of access restriction of vehicular flow approaching a node at each signal cycle. The node alternatively allocates green time among conflicting traffic movements seeking use of the same physical space. The inspection is valid not only in each signal cycle but equally in an observed time period for studying traffic phenomena. A similar restriction inequality on node capacity also exits in unsignalized intersections, but the alternation of access right is no longer prearranged rather it is adaptive: the advance is generally allowed for the early arrival.

#### **3. NETWORK CAPACITY AND RELIABILITY**

Network reserve capacity is here defined as the largest multiplier applied to a base origin-destination demand matrix that can be allocated to a transportation network in a user optimal way without violating the node capacities. It can be mathematically stated as follows:

 $\forall n$ 

max  $\mu$ 

subject to 
$$\sum_{i} \max_{a} \left\{ x_a(\mu \mathbf{q}) / c_a \middle| a \in A_a^i \right\} \le 1,$$

: equilibrium flow on arc  $a \in A$  $x_a(\mu \mathbf{q})$ 

: capacity of link  $a \in A$  $C_a$ 

: OD demand multiplier μ

: base OD demand vector q

 $\mu q$  is called the scaled demand, which is the base OD demands q scaled by  $\mu$ . A network capacity problem under link capacity constraints (Chen, 1999) can be formulated if the node capacity constraints are substituted by the following inequalities:

 $x_a(\mu \mathbf{q}) \leq c_a, \ \forall a$ 

Route choice behaviors are explicitly considered in equilibrium constraints, which bound equilibrium flows below their corresponding capacities. The pattern of equilibrium link flow is obtained by solving the following standard user-optimal traffic assignment problem:

minimize 
$$\sum_{a} \int_{0}^{x_{a}} t_{a}(x, c_{a}) dx$$
  
subject to  $\sum_{a} f_{r}^{w} = \mu \hat{q}_{w} \quad \forall w$   
 $x_{a} = \sum_{w} \sum_{r} f_{r}^{w} \delta_{ar}^{w} \quad \forall a$   
 $f_{r}^{w} \ge 0, \quad \forall r, w$   
 $W$  : set of OD pairs in the network  
 $R^{w}$  : set of routes between OD pair,  $w \in W$ 

 $t_a(x_a, c_a)$ : travel time on link,  $a \in A$ 

: demand between OD pair,  $w \in W$  $\hat{q}_w$ 

 $f_r^w$ : traffic value on route,  $r \in R^w$ 

 $\delta^w_{ar}$ : 1 if link a is included in route r, 0 otherwise The problem of computing multiplier,  $\mu$ , is treated as a bi-level programming problem. At the upper level link use proportions are used as the input, or equivalently, link flows, which are the output of standard user equilibrium assignment enforced at lower level. In consequence, route choice behavior and congestion effects are explicitly considered by the lower-level problem while the upper level problem determines the maximum OD matrix multiplier subject to the capacity constraints. As the scaled demand approaches the network capacity, equilibrium constraints will have a substantial effect on the distribution of traffic flow and on the network reserve capacity. Since the upper level problem has only one decision variable, it could be

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handled as a parameter at the lower level issue. And hence, the overall problem can be solved as a singular optimization, in which the multiplier,  $\mu$ , is properly adjusted until at least one of the equilibrium element flow approaching its upper bound, node or link capacity. Then, if,  $\mu > 1$ , the network has reserve capacity amounting to  $100(\mu - 1)$  percent of the base OD matrix, and if,  $\mu < 1$ , the network is overloaded by  $100(1-\mu)$  percent of the base OD matrix.

Capacity reliability is calculated as a probability that the maximum OD flow is greater than or equal to a required demand level when the capacity of links is subject to random variation. With the concept of reserve capacity, it can be given as

 $R(\mu^0) = P(\mu \ge \mu^0)$ 

The probability predicts how reliable the network with degraded links can accommodate a given demand level  $\mu_0$ . The system is 100% reliable when there is no demand, and 0% reliable when the demand is infinite. The employment of reserve capacity has provided a feasible approach to estimate network capacity reliability incorporating route choice behavior.

## **4.** NUMERICAL SIMULATION AND ANALYSIS

The numerical results of network capacity reliability are presented here with the use of Monte Carlo method. The test network, and base demand as well as link performances are the same as those (Inoue, 1986). The link travel time is estimated by the standard BPR function.

In the absence of link degradation data, a uniform distribution with upper bound is assumed to generate the random capacities of all links. When the capacity of every link is fixed at the upper bound equal to its capacity, the largest multiplier is one, which means that the current network capacity is just enough to accommodate the base demand, corresponding to non-degraded state. All the measures of capacity reliability are calculated from 5000 Monte Carlo simulations.

Figure 1 or 2 shows how the capacity reliability of network under link or node capacity constraints is variable versus the increase in traffic demand level when the variation width of each link capacity equal to .4 or .86 times its capacity). Under node capacity constraints, the network is 100% reliable in Figure 1 for lower demand levels up to 15% of the base demand, while under link capacity constraints the reliability of 100% will last until to 30% of



Figure 3: Reliability for various width of link capacity variation

the base demand. As the demand level increases, both measures of capacity reliability decline and finally fails. Reliability is rather different from vary visual points, and that the measure in node capacity constraints is conservative relative to that in link capacity constraints. There is no accident because the shortage of element throughput is usually occurred in the intersections rather than road segments over the street network. The reliability should therefore be concerned also under node constraints when network performance is studied.

The reliability measures in Figure 2 shows the same propensity like that in Figure 1, but the degradation of reliabilities are rapid when larger reduction in link capacities is specified. Note that the two reliability curves are coming closely, or say, the differences of the two measures becomes near when the amplitude of link capacity variation grows.

Figure 3 shows the reliability measures of demand level .45 for various reduction degrees of link capacities. The curve indicates that the variation width ( $\times$  certain constant) in link capacities has significant impact on the reliability performance of the network, especially on the measure defined under node capacities.

# **5.** CONCLUSIONS

A reliability measure of network capacity under node capacities has been introduced using the concept of network reserve capacity, and compared with that based on link capacities. The measure incorporating node capacities suggested should be considered in reliability analysis on road network. Note that the assumption that every OD pair will have a uniform growth or decline in its OD demand is preserved, whereas relaxing this limitation can yield pictures regarding the spatial distribution of the demand pattern reflect to non-uniform change, which can be especially useful in individual zone land-use development plans.

#### REFERENCES

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