Discrete Element Method for the Dynamic Buckling Analysis of Frames

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1. Introduction

When a slender structure of a frame type is subjected to a dynamic load of high intensity, e.g. earthquake or impulse loading it may experience buckling. On the other hand, with the rapidly changing loading conditions pertinent to dynamic loading, the state of unstable equilibrium may last only for a very short time, and thus not be dangerous in a sense that large displacements damaging the structure cannot occur instantaneously. Clearly, in terms of design it is not economic to prevent buckling completely, so simulation tools are needed to investigate the dynamic behavior including buckling and post buckling effects. To this end, new Discrete Element Method (DEM) approach is proposed allowing detailed investigation of buckling and post buckling behaviour in real time.

2. Method of analysis

The structural mass is lumped to the nodes. Rotational inertia is also assigned to nodes. Each mass stands for a discrete element in terms of the DEM. External loading may be modelled as an extra mass added to appropriate nodes. Alternatively, prescribed force, acceleration and velocity time histories may be applied to loaded nodes. The structural members between nodes are represented by springs with initial axial stiffness directly calculable from the axial stiffness of the member, and bending and shear springs of Bernoulli type. Supports are defined by specifying motion restraints to some elements as appropriate. Material nonlinearity can be incorporated by a suitable choice of the axial load- deformation curve for the springs. Elastic-plastic hysteretic model with strain stiffening was used for modelling steel. Currently, only plastic deformations arising from axial forces are incorporated in the computational program. In fact, for slender struts and for the purpose of this research elastic material model is sufficient. Geometrical nonlinearity is inherent to the solution procedure and no special provisions are necessary to account for it. The solution procedure is essentially dynamic relaxation, resulting in analyses being always dynamic. Outline of the solution algorithm is shown in Fig. 1 for an analysis of run time T. The forces and moments developing in the axial, shear and flexural springs adjacent to an element contribute to the driving force and moment of this element Analysis is always done in the time domain. Details on the mathematical treatment for the method are given in [1].

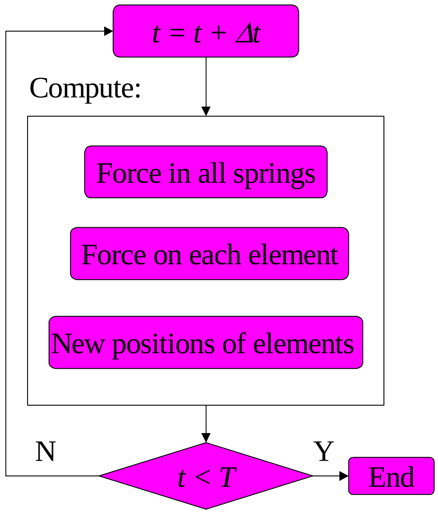

Fig. 1. Solution algorithm

3. Verification for accuracy

A 10m long beam fixed at both ends was modelled by eleven elements connected by ten springs. Properties Young's modulus $E = 2.1\text{x}10^8$ kN/m^2, area A=1x10^{-3} m^2 and second moment of area $I = 2.5\text{x}10^{-6}$ m^4 were assumed. Linear elastic material was used throughout. The deformed shape due to buckling as well as the set-up for the analysis is shown in Fig. 2. In order to make buckling possible some initial imperfection is needed. This was introduced by letting the beam assume it deflected shape under self-weight, which resulted in initial deflection of 0.047m at midspan. Further on, a gradual forced displacement (by constant velocity of 0.02cm/s) was applied to one of the supports resulting in gradual increase of the compressive force in the beam. The reaction force in one of the supports was monitored and its plot is

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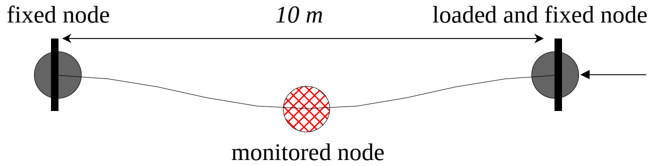

Fig. 2. Set-up for Euler buckling analysis

given in Fig. 3. The positive portion of the plot is due to the tensile force in the beam due to self-weight deformation. As forced displacement is input this is quickly cancelled out. Then the initial almost linear rise of the reaction force gradually becomes strongly non-linear approaching asymptotically the theoretical value of the Euler buckling load given for this case by the formula $N_{cr} = 4\pi^2 EI/L^2$.

4. Dynamic buckling behavior examples

In order to investigate possible dynamic effects a set of analyses was performed on the same structure with loading rates 4, 8 and 12cm/s. The results are shown in Fig. 4, from which it is immediately obvious that dynamic effects start to appear. Furthermore, for sufficiently large loading rates we observe that the structure is capable of momentarily sustaining loads larger than the Euler' limit (shown in the figure as a straight line). The reason for this is that due to inertia and damping it takes some time for the structure to respond to the loading, so for high loading rates a "dynamic stiffening" effect is observed. The proposed analysis procedure is capable of simulating this effect. Another set of buckling analyses was carried on the same structure with initial imperfection now being introduced as geometrical lack of fit. To this end, the whole structure was slightly rotated counter-clockwise around its left support to a position where its right support was 1cm higher than the left one, giving span/imperfection ratio of 1000. Loading was applied horizontally as before. The results are shown in Fig. 5. The dynamic stiffening is much stronger for the same loading rates for two reasons; first, the initial imperfection here is smaller, and second, the shape of the structure after initial imperfection is introduced coincides with the buckling mode shape for the first set of analyses, thus allowing buckling to proceed smoothly.

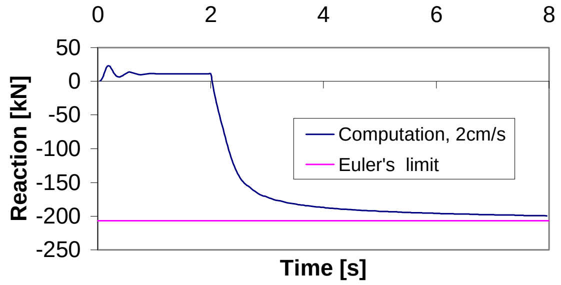

Fig. 3. "Static" loading

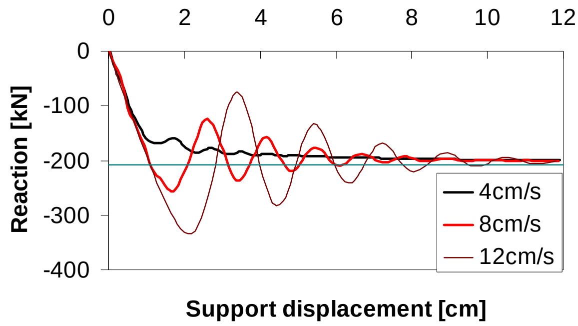

Fig. 4. Imperfection due to self-weight

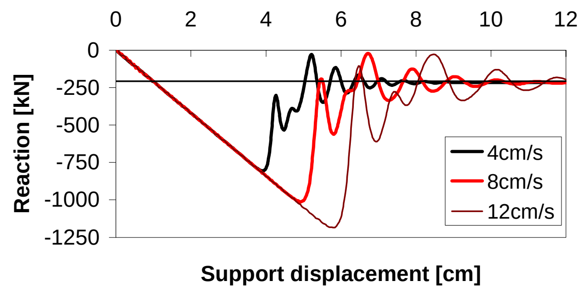

Fig. 5. Geometrical imperfection

5. Conclusions

A direct time domain method for nonlinear analysis of frame structures was formulated, its accuracy tested, and its ability to simulate buckling and post buckling behavior of frame structures demonstrated. Unlike traditional methods, there is not need to assemble a stiffness matrix, so singularity problems at buckling are avoided. Special provisions need not be made for geometrical nonlinearity. As shown by the two examples, it is possible to investigate the buckling phenomenon as a process rather than a state. It is thus possible to derive detailed information about the behavior depending on the type and magnitude of the initial imperfections. This is very important since as demonstrated the response can differ significantly according to the type of imperfections.

References:

1. Ivanov R. "Failure analysis of structures by the three dimensional discrete element method", 2001, Ph.D. Thesis, University of Kobe, pp. 104-105