Identification of Impact Force Using Laser Doppler Vibrometer (LDV)

University of Tokyo Member Sakda CHAIWORAWITKUL
University of Tokyo Member Masato ABE
University of Tokyo Fellow Yozo FUJINO

1. INTRODUCTION Accurate identification of input forces in a structure subject to operational excitations is an important issue from the aspects of design, control and diagnosis of the system. Traditional devices for recording structural responses, e.g. strain gauges were unreliable for this purpose because they tend to distort dynamic characteristics of the structures, especially damping, and hence yields inaccurate force predictions [1]. Thus, a more reliable, non-contact device, laser doppler vibrometer (LDV) is applied for the purpose. As such, the objective of this study is to develop a method for recovery of both magnitude and location of impact force using LDV in a clamped-free steel plate. The well-known Eigenvalue Realization Algorithm (ERA) [2] is applied in modal parameters extraction. Then, the force recovery equation is solved employing the pseudo-inverse technique [3].

Table 1 Characteristics of LDV

Laser type	He-Ne Laser
Wave length	633 nm
Laser output/class	2mw/ 2A
Possible measurement distance	30m
Resolution	0.5μm/sec
Measurement frequency range	0-35 kHz
Laser irradiation range	-15-15°

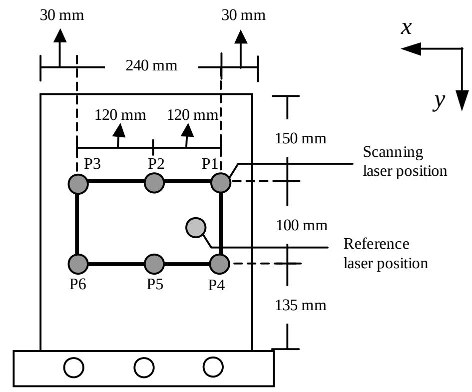
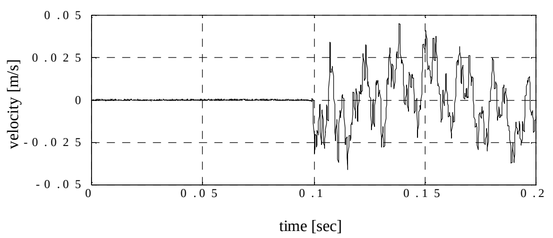
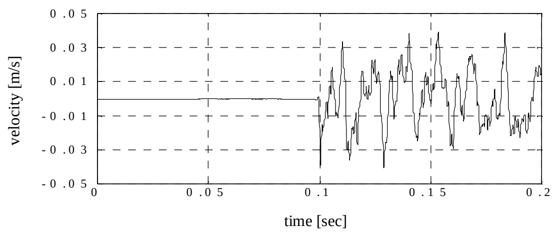


Fig. 1 Experimental Setup in Impact Identification



(a) Scanning Laser



(b) Reference Laser

Fig. 2 Measured Responses

2. PROBLEM FORMATION Assuming modal expansion, orthogonality and proportional damping, the equation of motion of a system can be decomposed into a series of modal equations in frequency domain using matrix notation as

$$[A]\{X(\omega)\}=\{B(\omega)\} \tag{1}$$

$$[A]=\begin{bmatrix} j\omega\left[\hat{\Phi}\right] & \{0\} \\ [K]-\omega^2 I+\omega[C] & -[\Phi]^T\{\delta\} \end{bmatrix}_{(2+p)\times(p+1)} \tag{2}$$

$$\{X(\omega)\}=\left\{\{Q(\omega)\}^T \quad F(\omega)\right\}^T_{(p+1)\times 1} \tag{3}$$

$$\{B(\omega)\}=\left\{\{\dot{U}(\omega)\}^T \quad \{0\}\right\}^T_{(p+2)\times 1} \tag{4}$$

$$K=diag\left[\omega_1^2 \quad \omega_2^2 \quad \ldots \quad \omega_{\mathrm{p}}^2\right] \tag{6}$$

$$C=diag\left[2\xi_1\omega_1 \quad 2\xi_2\omega_2 \quad \ldots \quad 2\xi_{\mathrm{p}}\omega_{\mathrm{p}}\right] \tag{7}$$

where Q, U and F denote fourier transform of modal amplitude, global displacement and impact force magnitude respectively. j denotes the imaginary unit and the superscript T refers to matrix transpose. ξ_i, and ω_i represent modal damping ratio and modal frequency of i-th mode, respectively. p denotes the number of lower modes used in modal expansion. $\hat{\Phi}$ is mode shape matrix at measuring points. δ denotes a location vector and can be defined as

$$\{\delta\}=\{0 \quad 0 \quad \ldots \quad 1 \quad \ldots \quad 0\}^T \tag{8}$$

That is, all the positions on the plate are set to be zero except the real location of the impact. It is important to note that the problem set as in equation (1) is always over-determined and can be solved in the least-squares sense employing pseudo-inverse technique, i.e.

$$\{X(\omega)\}=[A]^+\{B(\omega)\} \tag{9}$$

where $[A]^+$ is the pseudo-inverse of matrix $[A]$ defined as follows

$$[A]^+\equiv\left([A]^T[A]\right)^{-1}[A]^T \tag{10}$$

3. EXPERIMENT A clamped-free steel plate (385×300×2 [mm]) was used as a specimen in this study. The experiment was carried out using sampling interval 0.0001 [sec]. Scanning and reference lasers were set to measure the response at points shown in Fig. 1. The impact was given to the steel plate by an impulse hammer at point P2 at t = 0.1 [sec].

Key word: Impact Identification, Inverse analysis, Laser Doppler Vibrometer, Eigenvalue Realization Algorithm (ERA)
Address: Hongo 7-3-1, Bunkyo-ku, Tokyo 113, Japan Tel: 03-5841-6099; Fax: 03-5841-7454

Fig. 3 shows the plot of velocity responses obtained in the experiment from both lasers. Subsequently, the plot of recorded impact is shown in Fig. 3.

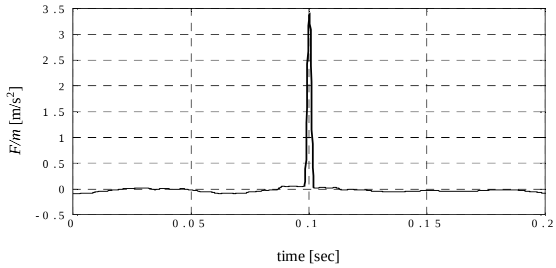


Fig. 3 Measured Force

Table 2 Modal Parameters Obtained From ERA

Mode	Frequency [Hz]	Damping ratio
1	10.94	0.0014
2	66.10	0.0013
3	112.2	0.0003
4	137.6	0.0001
5	191.0	0.0002
6	233.0	0.0001
7	369.6	0.0001

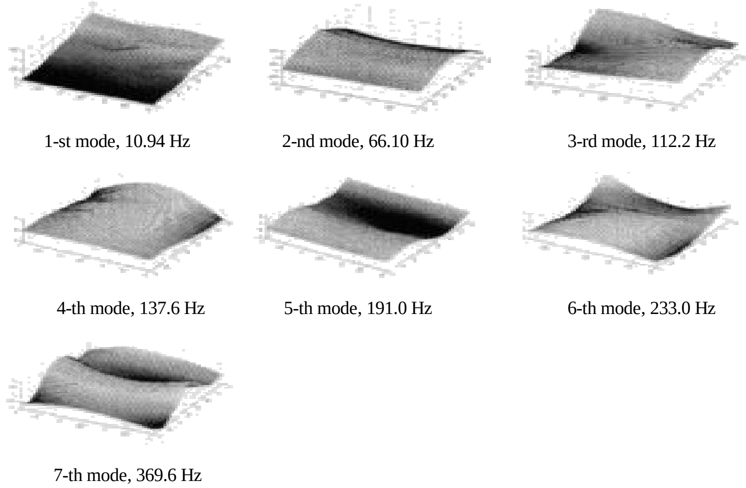


Fig. 4 Mode Shapes Obtained From ERA

4. VERIFICATION Calculation was performed using modal parameters obtained from ERA (see [2]). Results of ERA and lower modes employed in the prediction of force are shown in Table 2. Fig. 4 illustrates mode shapes obtained from ERA. Adapting these data and using method described in section 2, identification result obtained is compared in Fig. 5. Reasonable agreement can be observed from the figure between experimental and computational results. Identification of impact location was carried out by plotting the maximum value of force identified at each point on the plate as in Fig. 6. Force obtained at location x = 0.15 [m], y = 0.15 [m] gives the maximum value, i.e. it is the actual location, while the "forces" obtained in other points are essentially noise. This result matches with the real position where the impact was developed in the experiment, i.e. point P2 in Fig. 1. An indicator for assessment of the identified results is introduced herein as follows:

$$e(\hat{f}) = \frac{\int_0^t \left| \hat{f}(\tau) - f(\tau) \right| d\tau}{\int_0^t \left| f(\tau) \right| d\tau} \cong \frac{\sum \left| \hat{f}(\tau) - f(\tau) \right| \Delta\tau}{\int_0^t \left| f(\tau) \right| d\tau} \tag{11}$$

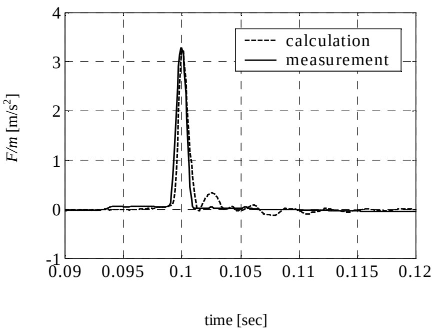


Fig. 5 Comparison of identified and measured forces

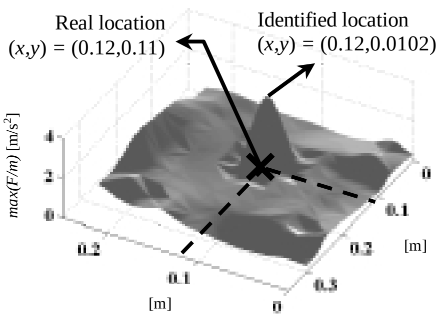


Fig. 6 Maximum value of identified force at each position on the plate

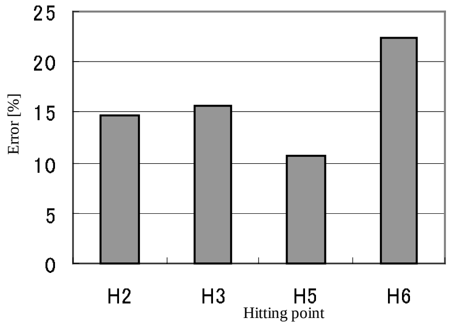


Fig. 7 Accuracy of prediction

The result of error indicator introduced in equation (11) is 14.71%. Validity of other cases in which impact locations were moved to point P3, P5, P6 are shown in Fig. (7) (each case is denoted as H2, H3, H5 and H6). Notice that the errors vary in the range 11-23%.

5. CONCLUSIONS In this study, a force identification method using LDV is introduced. Although measurement using LDV can be performed synchronously merely 2 points, the problem is still over-determined as described in section 2. Calculation errors vary between 16 to 24 percents, however this can be improved by adapting more number of modes into calculation because truncation of mode number in modal expansion leads to only an approximation results and hence causes in lower accuracy of prediction.

REFERENCE

1) Kammer, D.C., 1998, "Input Force Reconstruction Using a Time Domain Technique", *Transactions of ASME*, Vol. 120, pp. 868-874.
2) Vumanh H., Kaito K., Abe M., and Fujino Y., 2001, "The Eigensystem Realization Algorithm For Ambient Vibration Measurement Using Laser Doppler Vibrometers", *American Control Conference*, 2001
3) Edward, R.D., 1998, *Random Processes for Image and Signal Processing*, IEEE Press, USA