Theoretical Analysis on Traffic Volume Risk in PFI Transport Projects

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Introduction

In PFI transport projects, how to share the traffic volume risk is the key factor to achieve a good Value for Money. However, only a few theoretical studies on the traffic volume risks under various contract mechanisms have so far been conducted. The aim of this paper is to show mathematically and numerically how the traffic volume risk is borne by the project company under some particular contract mechanisms.

Scope, Framework and Arrangements of Analysis

Among numbers of types of contract mechanisms, the four types shown in **Table 1** are analysed and compared. They are classified according to linear/non-linear annual revenue structure and fixed/variable concession period. The annual revenue functions (ARFs), the relationships between traffic volume (*Q*) and annual revenue (*R*), are illustrated in **Figure 1**. The shadow toll with band system was applied to the first eight DBFO road projects and it enables the ARF to be non-linear (piecewise linear). The traditional BOT projects and DBFO projects have fixed concession period. Toll crossing projects adopt variable concession period varying with toll revenue and having maximum period. Two types of trigger for termination, the fixed present value and the fixed nominal value, are analysed.

The arrangements of the analysis are as follows. First, the wholelife revenue functions (WRFs), the present value of revenue (*PVR*) throughout the concession period as a function of traffic volume, are mathematically derived. For simplicity assume that the traffic volume is constant throughout the concession period and that the only uncertainty is the initial traffic volume. The profile of the traffic volume is illustrated in **Figure 2**. The concession period (*T*) may be changed according to the toll revenue. All other risks incorporated in the project are neglected in this analysis.

Next, the cumulative distributions of *PVR* are calculated by using Monte Carlo simulation to compare the traffic volume risk under each contract mechanisms.

Table 1　Comparison of Various Contract Mechanisms

Type of Contract Mechanism	Annual Revenue	Concession Period	Examples
Traditional Build Operate Transfer (**TBOT**)	Linear	Fixed	Most of BOT road projects
Shadow Toll with Band System (**STBS**)	Non-linear	Fixed	1st 8 DBFO road projects
Fixed Present Value of Revenue (**FPVR**)	Linear	Variable with Maximum	Dartford 2nd Severn, Skye
Fixed Nominal Value of Revenue (**FNVR**)	Linear	Variable with Maximum	

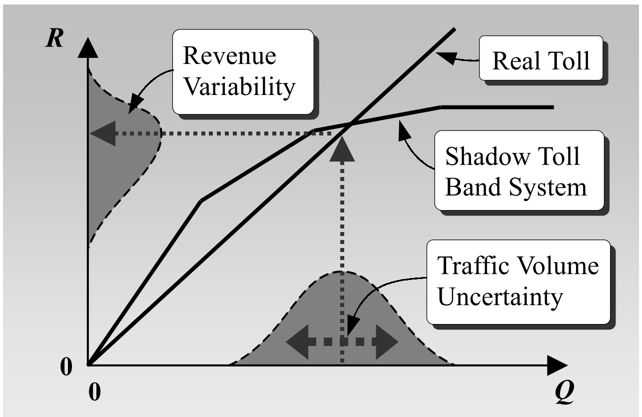

Figure 1　Annual Revenue Functions

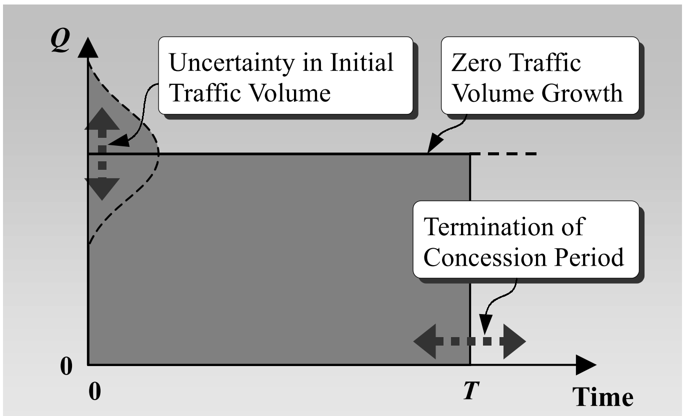

Figure 2　Assumed Traffic Volume Profile

Key words: PFI (Private Finance Initiative), Traffic Volume Risk, Shadow Toll, Concession Period, Monte Carlo Simulation
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Deriving Wholelife Revenue Functions (WRFs)

WRF is expressed as the present value of revenue (PVR):

$$PVR = \int_0^T Re^{-rt}\,dt = \frac{1-e^{-rT}}{r}R \qquad (1)$$

where r is the discount rate.

Under **TBOT**, $T = const.$ and $R = PQ$. Therefore,

$$PVR = aPQ \qquad (2)$$

where P is the price (toll), a is a constant.

Under **STBS**, $T = const.$ and $R = R(Q)$. Therefore,

$$PVR = bR(Q) \qquad (3)$$

where b is a constant, $R(Q)$ is a revenue function.

Under **FPVR**, if the traffic volume is higher,

$$PVR = \overline{PVR} \qquad (4)$$

where $\overline{PVR}$ is the pre-determined PVR.

If the traffic volume is lower,

$$PVR = \frac{1-e^{-rT_{max}}}{r}PQ \qquad (5)$$

where T_{max} is the maximum concession period.

The traffic volume that is the border is

$$Q = \frac{r\overline{PVR}}{P\left(1-e^{-rT_{max}}\right)}. \qquad (6)$$

Under **FNVR**, if the traffic volume is higher,

$$PVR = \frac{1-e^{-\frac{r\overline{NVR}}{PQ}}}{r}PQ \qquad (7)$$

where $\overline{NVR}$ is the pre-determined nominal value of revenue.

If the traffic volume is lower, the PVR is identical to that under **FPVR**. The traffic volume that is the border is

$$Q = \frac{\overline{NVR}}{PT_{max}}. \qquad (8)$$

These derived WRFs are illustrated in **Figure 3**.

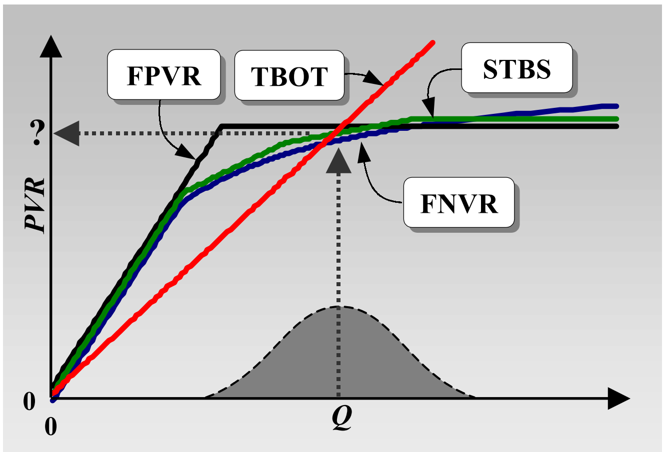

Figure 3　Wholelife Revenue Functions

Numerical Simulation

Detail of simulation is omitted here but the relationships between WRFs and the traffic volume are nearly what are shown in **Figure 3**. Normal distribution is assumed for the initial traffic volume. Calculated $PVRs$ were normalised so that the expected values would be 1.

The statistical results of the simulation are summarised in **Table 2**. The remarkable points are that the standard deviation under **TBOT** is large and the value with 5% confidence under **FPVR** is high. The values under **STBS** and **FNVR** lie between that of **FPVR** and that of **TBOT**. This tendency can be read from the cumulative distributions of PVR depicted in **Figure 4**. The other point to note is that, under **STBS**, **FPVR** and **FNVR**, the maximums are capped and the feet of lower PVR are not sufficiently capped.

Conclusion

The traffic volume risks under some contract mechanisms were analysed mathematically and numerically. We could confirm that the variable concession period and the non-linear shadow toll reduce the traffic volume risk compared to traditional BOT.

It should be noted that the results above depend on the assumptions and the parameters. Other important factors are also omitted here. Much still remains to be done to establish a general theory of sharing traffic volume risk.

Table 2　Statistical Results of Simulation

	TBOT	STBS	FPVR	FNVR
Mean	1.0000	1.0000	1.0000	1.0000
Standard Deviation	0.2502	0.0784	0.0553	0.0816
5% Confidence	0.5875	0.8539	0.9919	0.8593

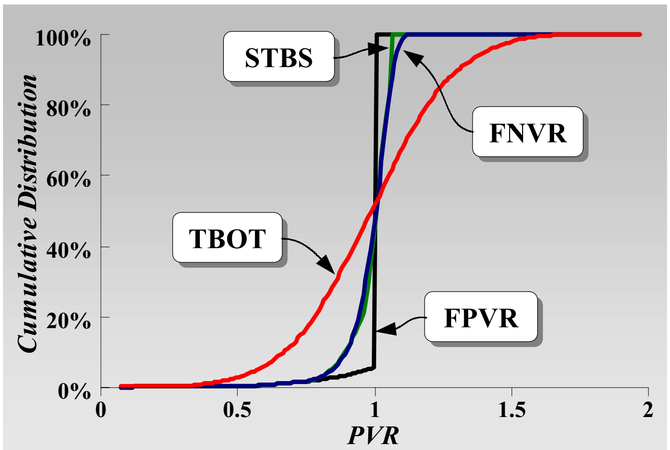

Figure 4　Cumulative Distribution of Revenue