# Network Equilibrium Model Considering Route-Choice Behaviors of Commuters under Uncertainty

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## 1. Introduction

The potential sources of disruption to road networks are numerous, like fluctuation of the travel demand and reduction of the link capacity due to incidents. Such disruption occurs unpredictably and thus results in uncertain traffic conditions of the network. An unexpected delay may introduce considerable loss to the transport users, and people thus always try to avoid an unreliable movement. Several studies have suggested that the variability and the consequent unpredictability of travel times are a significant factor in the choice of transport mode and route (see for instance Parkhurst *et al.*, 1992). However, little research has been done on modeling this risk-averse behavior in route choices. This paper attempts to incorporate the theory of decision-makings under uncertainty into network equilibrium models. It is expected that this approach may better describe the real route choice behaviors of travelers and thus provide better predictions of traffic conditions.

## 2. Risk-averse behavior in route choices

Risk aversion is the most generally observed attitude toward risk for uncertain events whose consequences are significant. The existence of insurance markets attests to this behavior. Similarly, because the penalty imposed to an unexpected later arrival is much higher than the bonus attached to an earlier arrival, travelers, specifically commuters are also risk averse in their choices of departure times and routes (Bruinsma *et al.*, 1999). In this paper, we only consider the route choice behavior in order to facilitate the presentation of our essential ideas.

An illustrative example is presented as follows to explain the risk-aversion in route choices. Consider the network in Fig. 1. Three routes are shown connecting a given origin to a given destination. Assume that each of the routes has one or two possible travel times in the morning peak period and their corresponding probabilities are also shown in Fig. 1.

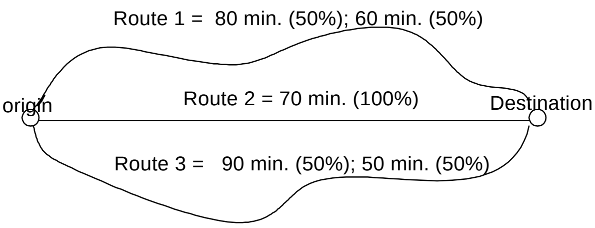


Fig. 1 Example network

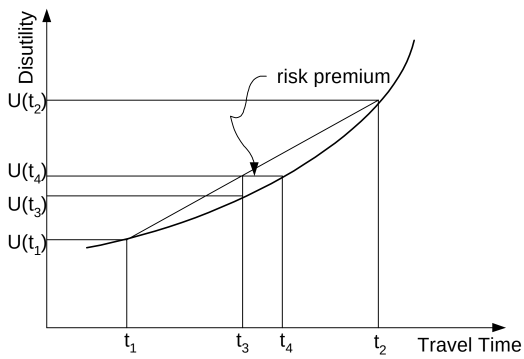


Fig. 2 Disutility curve and risk premium

Although the average travel times of these routes are the same, most commuters are observed to prefer route 2 to route 1 and then prefer route 1 to route 3 after a careful analysis. Such risk-averse behavior can be examined by the expected utility theory for decision-makings under uncertainty, where the concavity of the utility function implies risk aversion. To be consistent with traditional network equilibrium models, we made some changes to the original utility function. Here, disutility is associated with the travel time and commuters were assumed to choose the route with the minimal expected disutility. Thus, the disutility curve $U(t)$ used to depict the avoiding attitude toward risk should be convex, shown in Fig. 2. In the illustrative example, route 3 has two values of travel time, illustrated in Fig. 2 as $t_1$ and $t_2$. The average travel time of route 3 is $0.5t_1 + 0.5t_2 = t_3$, same as that of route 2. However, because commuters are risk-averse, the disutility of route 3 is $0.5U(t_1) + 0.5U(t_2) = U(t_4)$, which is larger than $U(t_3)$. That is the reason why commuters prefer route 2 to route 3. Furthermore, commuters would be indifferent between the

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route 3 and a link with a sure travel time of $t_4$. The difference between the certainty equivalent of the link travel time $t_4$ and the expected value $t_3$ is the risk premium, which is a common measure for the magnitude of deviation from risk-neutral behavior. A set of utility functions is available in the theory of decision-makings for different degree of risk aversion. Hence, the disutility function could be determined for a particular study after calibrations.

**3. Model formulation**

In this section, a network equilibrium model is proposed with consideration of the above risk-averse behaviors in route choices. The following basic assumptions and considerations were. First of all, we considered the link travel time as a random variable, which followed the normal distribution $N(\mu_a, \beta_a\mu_a)$, say, $t_a \sim N(\mu_a, \beta_a\mu_a)$, where $\beta_a$ is a link-related constant and $\mu_a$ is the average link travel time estimated by the traditional link travel time functions, e.g. the BPR function or Davidson functions. It is noted that this consideration has been conformed by previous studies (Taylor, 1999). Secondly, we further assumed that commuters knew well about the possible variations of their commuting times from the previous experiences. Although this assumption is restrictive and will be loosen in later research, a recent traveler behavior survey (De Palma and Rochat, 1999) found that commuters did have an in-depth knowledge of the network and thus showed high level of flexibility in their route choices.

With the above assumptions and considerations, the link expected disutility for risk-averse commuters could be formulated as

$$E_a(U) = \int \left(N(\mu_a(x_a), \beta_a\mu_a(x_a)\right) \cdot U(t_a)dt_a \qquad (1)$$

It can be proved that $E_a(U)$ is convex with respect to the link traffic volume $x_a$ provided that the disutility function and the link travel time function are convex. As assumed above, commuters would choose the route with the minimal expected disutility. Consequently, *a network equilibrium could be reached, in which no commuter can decrease his expected disutility by unilaterally changing routes*. In other words, in the equilibrium condition, commuters are indifferent with all the feasible none-zero-traffic routes.

Let $x_a^*$ be the user equilibrium link flows and $E_a(U^*)$ the corresponding link expected disuitlity. Then, under network equilibrium assignment the following link-based variational inequality holds:

$$\sum_a E_a(U^*)(x_a - x_a^*) \geq 0 \qquad (2)$$

where $x_a$ is any other link traffic flow in the feasible region defined by the flow conservation constraint and the definitional constraint.

It is known that the convexity of expected disutility function (1) ensures the existence and uniqueness of an optimal solution to the variational inequality problem (2). That is to say, the network equilibrium could be obtained by solving problem (2). The solution algorithm could be derived from minor modifications to the traditional traffic assignment algorithms, such as Frank-Wolfe, MSA, that $E_a(U)$ is determined by numeral integration or Monte Carlo simulation in each iterative.

**4. Discussion**

It is noted that if the disutility function takes a liner form, the expected disutility derived from Eq. (1) will be expected travel time, the solution to problem (2) will be exactly the traditional user equilibrium. The liner form of disutility function implies risk-neutral behaviors in route choices. Therefore, it was concluded that the traditional user equilibrium model implicitly treats the commuters as risk-neutrality and is only a special case of problem (2). Future research work may extend the proposed model to imperfect information about the variations of travel times, and apply the models into reliability analysis of road networks.

**Reference**

Bruinsa, F.R., Rietveld, P. and Van Vuuren, D.J. (1999) Unreliability in public transport chains, 8th WCTR Proceedings, Vol. 1, 359-372.

De Palma, Andre and Rochat, Denis (1999) Understanding individual travel decisions: results from a commuters survey in Geneva, Transportation, Vol. 26, 263-281.

Parkhurst, G., Kenny, F. and Goodwin, P. (1992) Quality attributes of a journey; their identification and impact on travelers. Transport Studies Unit, University of Oxford.

Taylor, M.A.P. (1999) Dense network traffic models, travel time reliability and traffic management. II: application to reliability, Journal of Advanced Transportation, Vol.33, No.2, 235-244.