Voids size distribution of soils related to the curves k_r- θ and S_u- θ

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1.Introduction: In this papers the voids distribution of two kind of soils is evaluated using the soil-water characteristic curve and the curve of relative permeability. These two curves, determined with Lab experiments enables the attainment of the voids size distribution of soils for the case the voids of soil are considered as pipes of different diameters aligned parallel to each other (Pipe model).

2.Pipe model of soils and its relation with the soil-water characteristic~relative permeability curves: Fig.1(a)-(b) are showing the pipe model of soils and the corresponding distribution of velocities and shear stress for a given pipe of radius *r*. The analysis of this diagram gives $k(r)=\gamma_w r^2/8u$ for the permeability of a pipe of radius *r* and $Q(r)=C_2 i r^4$ *for the rate of flow, with* $C_2=\pi\gamma_w/8u$. If *M* is the number of pipes per unit area per unit length, the number of pipes per unit area *m* between two radius r_1 and r_2 is

$$m(r)=\int_{r_1}^{r_2} p\ r^2 M(r)dr \qquad (1)$$

The capillary rise or S_u of a pipe of radius *r* is $h=C_1/r$, C_1 is a constant that involves surface tension, α , γ_w. On the other hand the relative coefficient of permeability k_r considering the summation of radius from r_{min} to *r* is

$$k_r(r)=\frac{\int_{r_{min}}^{r} r^4 M(r)dr}{\int_{r_{min}}^{r_{max}} r^4 M(r)dr} \qquad (2)$$

Fig.2 shows the above explained *M(r)* function, θ_r in the curves of Suction and relative permeability is the result of integration of all pipes from r_{min} to *r*, $k_r(r)$ also involves all pipes from r_{min} to *r* and $S_u(r)$ is suction for the pipe of radius *r*.

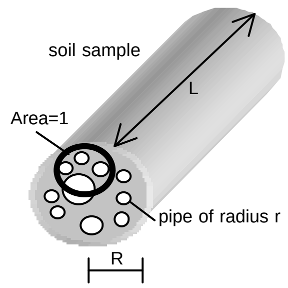

Fig.1(a). Pipe model.

3.Mathematical formulation of the function M(r)

We suppose now that the function *M(r)* is discontinuous in order to make mathematical formulation easy to understand. In other words *M(r)* is simplified to a function *n(r)* (number of pipes per unit area) where n_j is the number of pipes per unit area for a radius $r=r_j$. Assuming that C_1 is known, the analysis of two consecutive pipes r_j~r_{j-1} gives

$$\mathbf{r_j}^2=C_1^2/h_j^2 \qquad \mathbf{n_j}=h_j^2(\theta_j-\theta_{j-1})/\pi\ C_1^2 \qquad (3)$$

Both h_j and θ_j are the coordinates of points on the soil-water characteristic curve. This points were obtained with laboratory tests. In this way it is possible to obtain the voids distribution of soils whenever the pipe model were applied to the soil. On the other hand, if we assume that C_2 is known we can write

$$\mathbf{n_j}=C_2(\theta_j-\theta_{j-1})^2/\pi^2 k_s(k_{rj}-k_{rj-1}) \text{ and }$$

$$\mathbf{r_j}^2=\pi k_s\ (k_{rj}-k_{rj-1})/C_2(\theta_j-\theta_{j-1}) \qquad (4)$$

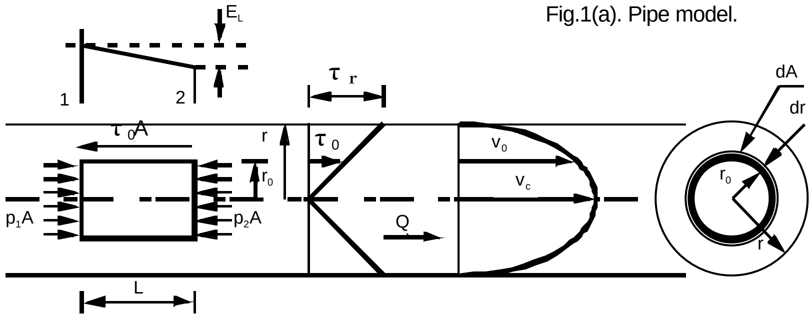

Fig.1(b). Distribution of vel. and shear stresses for a pipe of radius r.

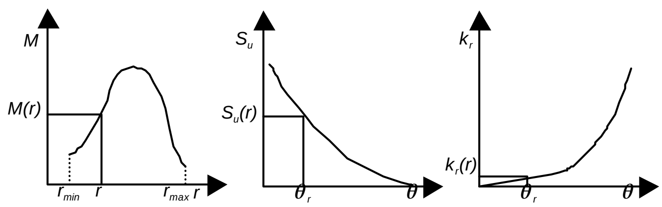

Fig.2. Function *M* (pipes/unit area/unit length) and k_r~S_u curves.

Key words: Pipe model, Soil-water characteristic curve, relative permeability, voids size distribution.

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Regardless of the curve from which the function $n=f(r)$ is obtained ,the curve of capillary rise or the curve of relative permeability, it should be the same for the same soil, for this purpose

$$C_1^2\, C_2 = \pi k_s h_j^2 (k_{rj} - k_{rj-1}) / (\theta_j - \theta_{j-1}) \qquad (5)$$

4.Calculation of discontinuous function n=f(r) for sand and masado soil: Fig.3-4 show the soil-water characteristic curve and the curve of relative permeability for sand and masado soil. Both curves were obtained by means of the Flux Method. According to the mathematical formulation explained before, the function $n=f(r)$ is calculated using the points of the curves of Fig.3-4. Results showed that calculated radius r_j are not in progression, then for some points of the discontinuous function $n=f(r)$, $r_j<r_{j-1}$. When $n=f(r)$ is obtained from the soil-water characteristic curve the origin of regression of the radius is the case when θ increases the values of suction increase. This illogical result is inadmissible concerning the pipe model proposed and must be corrected making $\theta_{j-1}=\theta_j$ in the curve of suction obtained with the Flux Method. When $n=f(r)$ is obtained from the curve of relative permeability increasing values of θ corresponds to increasing values of k_r. At first sight this result is logic, however the regressive values of radius are not determined by the term k_r-k_{r-1} but the term k_r-k_{r-1}/ θ_j-θ_{j-1}. For this case regression must be corrected adjusting the measured values of θ or k_r during the Flux Method. The same conclusion was learned when the objected soil was Masado soil. Figures 5(a)(b)(c) and 6(a)(b)(c) shows the adjusted function $n=f(r)$ (adjustment of regressive values of radius), in curves (a) the values of θ are corrected, keeping the values of k_r constant, in (b) the values of k_r are corrected keeping the values of θ constant, finally in (c) the capillary rise h (S_u) is corrected in order to avoid the regressive values of radius. This figures also show the value of saturated permeability k_s calculated using the function $n=f(r)$. Calculation of the $n=f(r)$ function was made considering C_1 for the case of a tube or radius r in which water rises but actually is not the real situation of the voids of soils. The same is for C_2 which is considered for water flowing through the conduit of Fig.1. This simplification makes the value of saturated permeability attained through the function $n=f(r)$ different for the saturated permeability attained from laboratory tests in some cases of Fig.5-6. In addition to that, critical points are not included in the calculation of the $n=f(r)$ function. For instance, the point of the soil-water characteristic curve $S_u=0$ is omitted as the radius becomes infinite.

5.Conclusions

From Fig.5-6 the smallest pipes are dominant when considering the term $n_j \pi r_j^2$,this may explained the flat portion of the k_r curve and shape of the soil-water characteristic curve. Contrarily, when considering the term n_jQ_j the largest pipes are dominant, which may explained the steep portion of the curve of relative permeability.

References: ENOKI, M(1999) Infiltration of rain water in slopes Proceedings of the 34th JAPAN NATIONAL CONFERENCE on Geotechnical engineering.

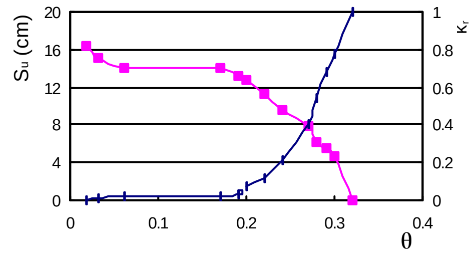

Fig.3. Curves k_r-S_u for sand.

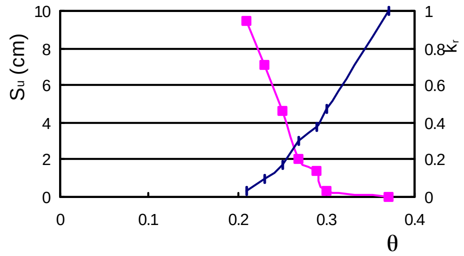

Fig.4. Curves k_r-S_u for Masado.

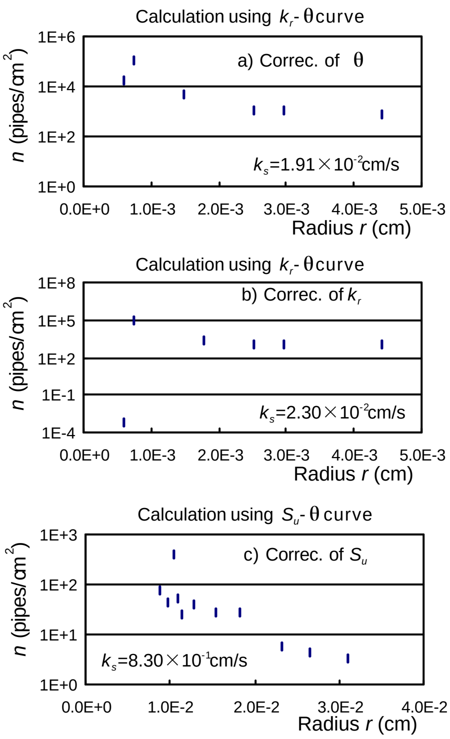

Fig.5. Calc. of the function n for sand.

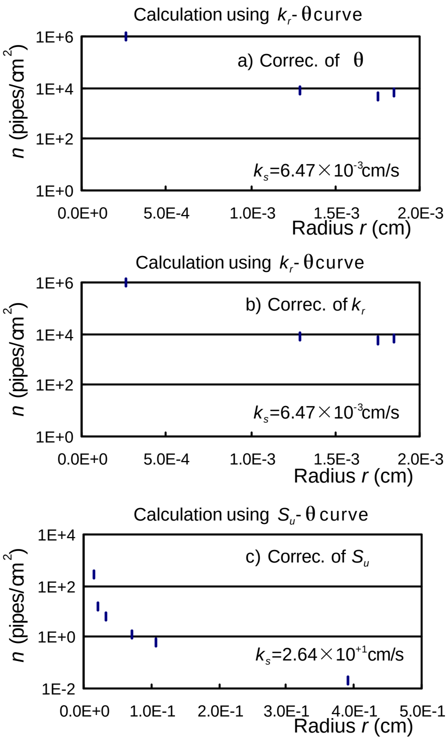

Fig.6. Calc. of the function n for masado.