3D FEM Analysis for Nonlinear Dynamic Response of Saturated Soil Layer Behind the Rigid Gravity Wall

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1. Introduction

In this paper, the dynamic response of the saturated soil layer behind the rigid wall was analyzed using 3D FEM. A consideration of the interaction between the granular soil and the pore water are taken into account by idealizing the saturated soil layer as a two-phase system. A nonlinear property of the granular soil is represented by the bounding surface plasticity model. In order to reduce the computing time as well as the computer memory requirement, the FEM formulation has been derived for using a parallel multiprocessor machine. For this purpose the Domain Decomposition Method (DDM) and Weighted Residual Method (WRM) were applied¹). Finally, an experimental model based on work of Kawamura² was chosen to show the accuracy a numerical result.

2. Governing Equation

Governing equations of motion of a saturated soil are derived based on following Biot's equations and the mass conservation, respectively³.

$$\sigma_{ij,i} + \rho g - \rho \ddot{u}_j - \rho_f \ddot{w}_j = 0 \tag{1.a}$$

$$-\frac{\dot{w}_j}{k_{ij}} = p_{,j} - \rho_f g + \rho_f \left(\ddot{u}_j + \frac{\ddot{w}_j}{f} \right)$$
(1.b)

$$\alpha \dot{\varepsilon}_{ii} = -\dot{w}_{i,i} - \frac{l}{Q}\dot{p} \tag{1.c}$$

in which σ_{ij} are the components of the total stress tensor for the saturated material, tensile normal stresses are assumed positive; p is the pore water pressure, compression is assumed positive; ρ , ρ_s and ρ_f are the mass density of the coupled system, the granular soil and the pore water, respectively; f is the porosity; g is the gravitational acceleration; k_{ij} are the effective permeability tensor; u_i and w_i are the displacement fields of the granular soil and the pore water, respectively. The superposed dot implies time derivative and (O_j) denotes the first order derivative with respect to coordinate x_j . A constant α represents the contact area of the soil particles and Q denotes the storage due to compressibility of the granular soil and the pore water. Eqs. 1.a and 1.b implies to a representative unit volume of the saturated soil layers and the pore water, respectively.

3. Constitutive Relation

The granular soil skeleton and the pore water have an individual material property, which follows the constitutive relation in Eq. 2 below.

$$\dot{\sigma}_{ij} = D_{ijkl}\dot{\varepsilon}_{kl} + \alpha Q\delta_{ij} \left(\alpha \delta_{kl}\dot{\varepsilon}_{kl} - \dot{\zeta}\right)$$
(2.a)

$$\dot{p} = -Q\left(\alpha\delta_{ij}\dot{\varepsilon}_{ij} - \dot{\zeta}\right) \tag{2.b}$$

in which the superposed dot implies the rate and δ_{ij} is the Kronecker's delta. The fourth rank tensor D_{ijkl} represents the components of elastoplastic tensor, including the effect of material non-linearity. The tensor ε_{ij} and ζ denote the strains of the granular soil and a volume change of the pore water, respectively.

$$\varepsilon_{ij} = \frac{1}{2} \left(u_{i,j} + u_{j,i} \right) \tag{3.a}$$

$$\zeta = -w_{i,i} \tag{3.b}$$

The constitutive model use to describe the stress-strain relationship for the granular soil is based on the radial mapping type of the bounding surface plasticity model within the framework of elastoplasticity and critical state of soil mechanics^{4,5)}. The bounding surface is expressed in a function of the invariant of effective stress *I*, *J*, *S* and 'Lode' angle φ . Compression normal is assumed positive. The component of elastoplastic tensor in Eq. 2.a, is specifically written as follow.

$$D_{ijkl} = E_{ijkl} - h(L) \frac{E_{ijmn} L_{mn} L_{pq} E_{pqkl}}{K_p + E_{abcd} L_{ab} L_{cd}}$$
(4.a)

$$L_{ij} = \nabla F = F_{\bar{J}} \delta_{ij} + \frac{F_{\bar{J}}}{2J} s_{ij} + \frac{\sqrt{3}}{2\cos 3\alpha} \frac{F_{,\varphi}}{\beta J} \left[\frac{s_{ik} s_{kj}}{J^2} - 3 \left(\frac{S}{J} \right)^3 \frac{s_{ij}}{2J} - \frac{2}{3} \delta_{ij} \right]$$
(4.b)

$$L = \frac{L_{pq}E_{pqkl}}{K_p + E_{abcd}L_{ab}L_{cd}}\dot{\varepsilon}_{kl}$$
(4.c)

where h(L) is heavystep function, h(L > 0) = 1 and $h(L \le 0) = 0$; a fourth rank tensor E_{ijkl} is the elastic moduli tensor, which is a function of the stress state. A vector L_{ij} and a constant L defines the loading direction in stress space and the loading index, respectively. L>0, L=0 and L<0 signs the plastic loading, neutral loading and unloading, respectively. F is an equation of the bounding surface plasticity model as reported in [4].

4. Solution Strategy

The Domain Decomposition Method (DDM) was used to separate the analysis of whole of domain into several non-overlapping subdomains through an interface problem. The interface problem was solved by an iterative solver Conjugate Gradient (CG) procedure with an interprocessor communication based on hypercube paradigm. Spatial discretization of the governing equations were carried out using Galerkin finite element approximation based on a Weighted Residual Method (WRM) over all subdomains⁶). If the series of a small step of the nonlinear responses are approximated linear, the equation of motion can be expressed in an incremental form. The solution of equation of motion is solved by the Newmark time integration scheme. A final equation is thus written as follow.

$$\sum_{s=l}^{N_s} \left[\hat{\boldsymbol{K}}_T^{(s)} \Delta \boldsymbol{d}_n^{(s)} \right] = \sum_{s=l}^{N_s} \left[\Delta \hat{\boldsymbol{F}}_n^{(s)} - \boldsymbol{B}^{(s)^T} \boldsymbol{\lambda}_n \right]$$
(5.a)

subject to continuity condition

$$\sum_{s=1}^{N_s} \boldsymbol{B}^{(s)} \Delta \boldsymbol{d}_n^{(s)} = 0$$
(5.b)

in which

$$\hat{\boldsymbol{K}}_{\boldsymbol{T}}^{(s)} = \frac{4}{\Delta t^2} \boldsymbol{M}^{(s)} + \frac{2}{\Delta t} \boldsymbol{C}^{(s)} + \boldsymbol{K}_{\boldsymbol{T}}^{(s)}$$
$$\Delta \hat{\boldsymbol{F}}_n^{(s)} = \Delta \boldsymbol{F}_n^{(s)} + \left(\frac{4}{\Delta t} \boldsymbol{M}^{(s)} + 2\boldsymbol{C}^{(s)}\right) \dot{\boldsymbol{d}}_{n-1}^{(s)} + 2\boldsymbol{M}^{(s)} \boldsymbol{\ddot{\boldsymbol{d}}}_{n-1}^{(s)}$$

The matrices $M^{(s)}$, $C^{(s)}$ and $K_T^{(s)}$ are the subdomain mass, damping and *tangent* stiffness matrices, respectively. Superscript (s) denotes the subdomain number, N_s is the number of subdomains, $[]^T$ denotes a transpose and the superposed dot implies time derivative. The rectangular matrix $B^{(s)}$ is signed Boolean matrix, which localizes the subdomain quantity to the subdomain interface. A vector λ is the traction force on the interface nodes. A vector $d^{(s)}$ is the subdomain displacement vector containing the displacement vectors of the granular soil skeleton and the pore water. The equation of motion (5) for defining the incremental displacement vector includes the effect of material nonlinearity. The modified Newton-Raphson procedure is used to solve the non-linearity condition.

Table 1. Material Properties

Property		Value
Slope of isotropic consolidation line	λ	0.005
Slope of elastic rebound line	κ	0.14
Slope of critical state line	Nc	1.72
Ratio of extension to compression	Ne/Nc	0.95
Value of parameter defining the ellipse 1	Rc	2.25
Value of parameter defining the hyperbola	Ac	0.001
Value of parameter defining the ellipse 2	Т	-0.1
Projection center parameter	С	0.0
Elastic nucleus parameter	S	1.0
Ratio of triaxial extension to compression	Re/Rc	1.0
Ratio of triaxial extension to compression	Ae/Ac	1.0
Shape of hardening parameter	hc	20.0
Ratio of triaxial extension to compression	he/hc	1.0
Void ratio	e	0.64
Poisson's ratio	ν	0.33
Bulk modulus of granular soil	Ks	3.7e5
Bulk modulus of pore water	Kf	2.1e4
Coefficient of permeability	k	0.012
Density of saturated soil	ρ	0.0016
Density of pore water	ρf	0.001

5. Numerical Example

The experimental data of Toyoura fine sand obtained by Kawamura has been used to validate the proposed procedure. The model with dimension of 2.00 m long, 1.00 m wide and 0.56 m high was vibrated on the shaking table equipment. A sinusoidal motion with the amplitude of acceleration 300 gals and frequency of 3 Hz was apllied as the horizontal base motion.

The experimental model mentioned above was analyzed using eight processors in Cray SGI Origin/2000 machine. The material properties used in analysis are listed in Table 1. The properties for defining the bounding surface configuration was taken based on these default values in reference (5), while other properties were experimentally defined. The dynamic responses of the pore water were picked up in several nodes in the left side of model. Comparison of the numerical result with the experimental data is shown in Fig. 1.



Fig.1 Comparison the numerical result with the experimental data

6. Conclusion

The accurate and efficient parallel 3D FEM procedures for solving the nonlinear dynamic response of the saturated layers have been developed. Comparison the numerical result with the experimental data shows the capability of the FEM analyses.

References

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