Multifractal Properties of Rainfall Observations: An Analysis of the Temporal Variation

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1. Introduction

Fractal and Multifractal analysis has gained popularity in recent years as a tool to understand temporal and spatial variation of physical fields at varied resolutions. Many geophysical fields including precipitation intensity show multiple scaling properties, which require the application of multifractal theories to analyze the scaling relationships. The *universal multifractal theory* uses the assumption that a field is a result of a multiplicative cascade process, in order to simplify the general multifractal theory so that it is possible to describe the scaling fields using a finite number of model parameters (Tessier, et al.,1993). This paper explains the application of the Probability Distribution Multiple Scaling (PDMS) method to analyze the temporal variations of rainfall.

2. Theory

(1) The Power Spectral Density Function

The first step in a multifractal analysis exercise would be to find the scaling regime of the observed measure. One method to do this is to analyze the measure in Fourier space, for the power spectral density function. Within the scaling regime the power spectral density $S(k)$ (defined as $S(k) = \lim_{T\to 0} \frac{1}{T}|Y(k,T)|^2$ where Fourier transform is denoted by $Y(k,T)$)relates to the wave number, k takes the following form.

$$S(k) \sim k^{-\beta} \qquad (1)$$

Hence, in region that representing an approximately linear segment on the power spectral density function on the log-log scale a multifractal field can be assumed to be scaling.

(2) Universal Multifractals

A field ϕ_λ is said to be *conserved* if the ensemble average of the field ($\langle\phi_\lambda\rangle$) is independent of the scale of interest, λ. According to the PDMS theory the scaling properties of a conserved multiscaling field ϕ_λ can be explained by the following equation (Tessier, et al.,1993):

$$P(\phi_\lambda \leq \lambda^\gamma) \propto \lambda^{-C(\gamma)} \qquad (2)$$

Where λ is the non-dimensional scale obtained by dividing the largest scale of interest T by the scale t. ϕ_λ is the field obtained by normalizing the original field by the ensemble mean of the filed. γ is a scaling exponent and $C(\gamma)$ is known as the *codimension function*. In *Universal Multifractals* the codimension function is defined by the following functional form:

$$C(\gamma) = C_1\left(\frac{\gamma}{C_1\alpha'} + \frac{1}{\alpha}\right)^{\alpha'} \quad \alpha \neq 1 \qquad (3)$$
$$\text{(for } 0 \leq \alpha \geq 2)$$

Where C_1 is the value of the codimension for the mean process. The multifractality parameter α, also known as Levy index, indicates the probability distribution involved in the underlying multiplicative cascade process. $\alpha = 0$ is for the unifractal case and values larger than zero indicates multifractal nature. The case $\alpha = 2$ corresponds to lognormal multifractals.

3. Methodology

The hourly rainfall data published by the Japanese Meteorological Agency was used in this analysis. Five rainfall series at gauging stations near Tokyo were analyzed. Each of the stations contained about 20 years of rainfall records. In order to achieve some degree of temporal homogeneity, only 100 days from each year, starting from April 1st was used in analysis.

The power spectral density function of each rainfall series was plotted with the wave number in log-log scale (See figure 1). The approximately straight-line portion of the spectral density function indicates the scaling regime. It was observed that the scaling is observed from 1 hr scale to

Table 1 Estimated Multifractal Parameters.

	C_1	α
Funabashi	0.3647	0.7607
Abiko	0.3869	0.7276
Chiba	0.5049	0.3507
Tokyo	0.3350	0.9857
Shin Kiba	0.3851	0.7264
Average	0.3953	0.7102
st.dev.	0.0647	0.2281

about 48 hr scale. After obtaining the scaling regime, the multifractal properties were analyzed in that interval. Figure 2 shows the plot of P against λ at various values of scaling exponent γ. Codimension function parameters (C_1 and α) were obtained from the slopes of the straight-line segment of the curves using a non-linear estimation method. Figure 3 shows an example for the estimated codimension functions. The scatter observed at low γ values may be attributed to the low measuring precision. Table 1 shows the multifractal parameter values for the stations.

4. Conclusions

Both, the analysis of spectral slopes and the linear segments in P vs. λ curves show that there exists a scaling regime extending from 1hr to approximately 2days resolution. The existence of this scaling provides a means to relate the rainfall distributions between various scales in that range. For example this study indicates that it is possible in principal, to obtain hourly rainfall distributions from those observed at daily resolutions. The values obtained for multifractal parameters (C_1 and α) indicate that the rainfall fields are definitely of multifractal nature, thus, the single fractal approaches may not adequately describe the variations in the process. The possibility of existence of well matching codimension curve indicates that it is possible to assume rainfall process as a multiplicative cascade process and to use the universal multifractal approach to create rainfall models to relate different temporal resolutions.

REFERENCES

1) Tessier, Y., Lovejoy, S. and Schertzer, D., Universal Multifractals: theory and observations for rain and clouds, Journal of Applied Meteorology, *2* 223-250,1993.

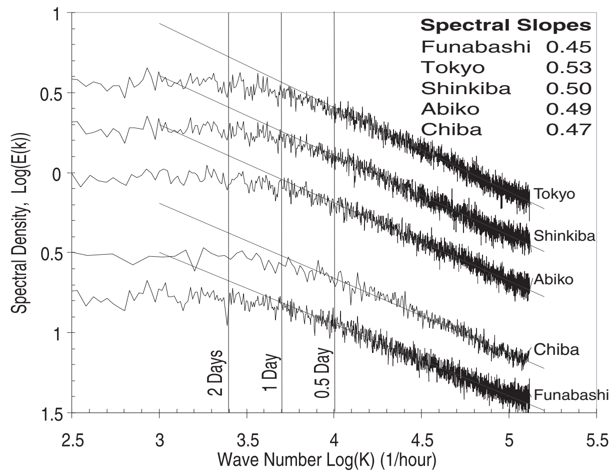

Fig. 1 Power spectral density plots in logarithmic scale. In many cases the scaling breaks at a scale around 1-2days. The data were averaged over logarithmically spaced intervals for plotting

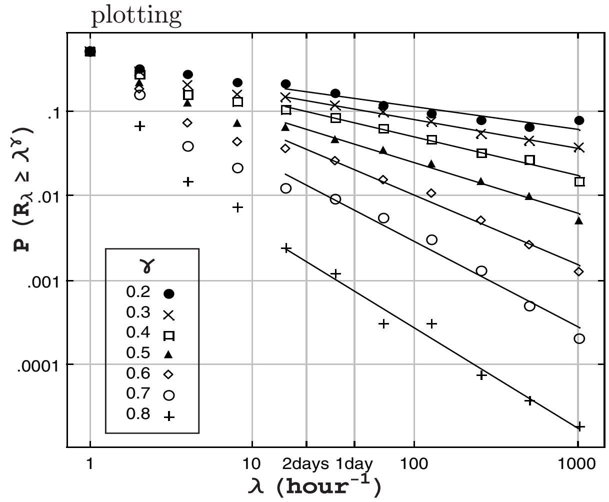

Fig. 2 The $P-\lambda$ curves for the Observations in Tokyo. Only a limited number of γ values are shown in the figure for clarity.

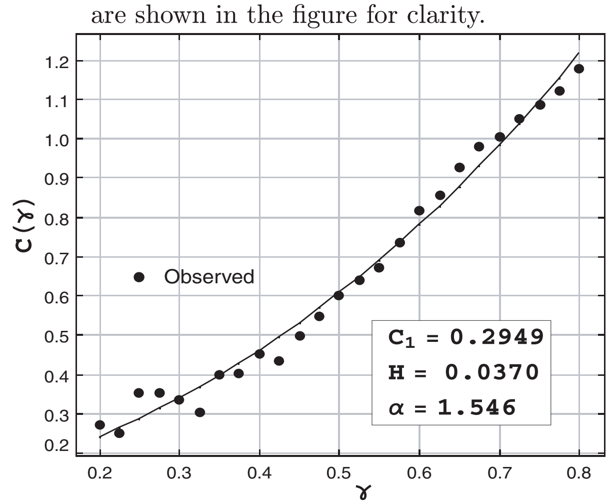

Fig. 3 The Codimension function estimated for Tokyo.