# FREE-SURFACE MOTION OF HOMOGENEOUS SOILS AS AFFECTED BY SATURATION<sup>1</sup>

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#### **1. INTRODUCTION**

A recent analysis of array observations has shown that water saturation of soils may strongly affect ground motion in vertical component (Yang & Sato 2000), suggesting the importance of saturation condition in the interpretation of field observations. In this paper, a study is presented of the saturation effect on the motion in both horizontal and vertical components at the free surface of soil media due to *SV*-wave incidence. The soil is modeled here as a partially water saturated porous material with only a small amount of air inclusions. In what follows a brief theoretical formulation is described, and selected numerical results are given to illustrate the influence of saturation on the surface motion in the two components.

#### 2. THEORETICAL FORMULATION

The typical case considered is when the degree of saturation is sufficiently high so that the air is embedded in pore water in the form of bubbles. For this special case the concept of homogeneous pore fluid may apply to the theory of two-phase porous media and the bulk modulus of the homogeneous fluid  $K_f$  approximately depends on the degree of saturation as (Yang & Sato 2000)

$$K_{f} = \frac{1}{\frac{1}{K_{w}} + \frac{1 - S_{r}}{p_{a}}}$$
(1)

in which  $K_w$  is the bulk modulus of pore water and  $p_a$  is absolute fluid pressure.

Based on the concept of homogeneous pore fluid and the theory for two-phase porous media, the governing equations for wave motion can be given as (Yang & Sato 2000)

$$\mu \nabla^2 \mathbf{u} + (\lambda + \alpha^2 M + \mu) \nabla e - \alpha M \nabla \zeta = \rho \ddot{\mathbf{u}} + \rho_f \ddot{\mathbf{w}}$$
<sup>(2)</sup>

$$\alpha M \nabla e - M \nabla \zeta = \rho_f \ddot{\mathbf{u}} + \frac{\rho_f}{n} \ddot{\mathbf{w}} + \frac{\eta}{k} \dot{\mathbf{w}}$$
(3)

where  $\alpha$  and M are parameters accounting for the compressibility of grains and fluid, they can be given as

$$\alpha = 1 - \frac{K_b}{K_s} \qquad M = \frac{{K_s}^2}{K_d - K_b} \qquad K_d = K_s [1 + n(\frac{K_s}{K_f} - 1)]$$
(4)

in which  $K_s$  and  $K_b$  are bulk moduli of solid grains and skeleton, respectively;  $K_f$  is the bulk modulus of pore fluid, it is related to the bulk modulus of pore water, absolute fluid pressure and degree of saturation as described in (1). The description of remained quantities in (3) and (4) can be found in Yang & Sato (1998).

The governing equations can be de-coupled into two pairs of equations that respectively describe the propagation of compressional waves (two types) and one shear wave (Yang & Sato 1998). In Fig. 1 the influence of saturation on the velocities of three types of waves in a typical sand is shown. Hence, for the boundary value problem under consideration (Fig. 2), there exist three reflected body waves (two types of P waves and one SV wave) due to the incident SV wave. The wave fields at the surface can be given following similar procedures as shown in Yang &Sato (1998). By enforcing the boundary conditions (free-stress and free-drainage surface), the reflection coefficients can be obtained as a function of degree of saturation, incident angle and frequency. The amplitudes of horizontal and vertical motion at the surface can be subsequently achieved based on these coefficients.

## 3. INFLUENCE OF SATURATION ON SURFACE MOTION IN TWO COMPONENTS

In what follows numerical results are given to illustrate the influence of saturation on horizontal and vertical components of motion at the surface. The sand with typical values as employed in the computation of velocity in Fig. 1 is used in the computation. Fig. 3 shows the displacement amplitudes of horizontal and vertical components of motion as a function of incident angle. Three cases of saturation are included to show the effect of saturation. In order to simultaneously depict the influence of incident angle, both the horizontal and vertical amplitudes are normalized by the amplitude due to vertically incident *SV* wave. For all cases of saturation under consideration, a reasonable feature is observed, that there are no vertical displacements at normal incidence and the displacements in both components vanish when the angle of incidence is 90 degrees. In addition, it is noted that the horizontal displacement is zero at the angle of 45 degrees. For any case of saturation, it is found that the horizontal

<sup>&</sup>lt;sup>1</sup> *Keywords:* partial saturation, ground motion, porous soil, wave propagation



Fig. 1 Effect of saturation on wave velocity in sand



Fig. 2 Model considered in this study



Fig. 3 Effect of saturation on horizontal and vertical motion



Fig. 4 Effect of saturation on displacement ratio V/H

components of inclined *SV* wave are in general smaller than those corresponding to vertical incidence, except in a narrow range of incident angles around which a peak forms. In particular, a considerable difference is noticed in this range between the complete saturation case and partial saturation case. For the case of incomplete saturation, the peak appears to be very sharp whereas the vertical component at this point forms a downward cusp.

The results for the ratios of vertical to horizontal components in the three cases of saturation are shown in Fig. 4. For all cases of saturation a singularity is found at the angle of 45 degrees because the horizontal displacement is zero at this point as seen before. A closer view of the ratios beyond the singularity is given in Fig. 4b and 4c, which indicates that a noticeable influence of saturation on the ratio exists, especially in the range of incident angles of 25 to 35 degrees and the range of 50 to 58 degrees, both ranges are in vicinity to the singularity.

### REFERENCES

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