# V-168

# A SIMPLIFIED MODEL FOR SNOW MELTING PAVEMENT SERPENTINE HEAT EXCHANGERS

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### 1. Introduction:

Pavement snow melting systems most often use a serpentine layout for the warm water pipes embedded below the surface of the pavement. It is essential that such systems achieve the most uniform pavement surface temperatures possible. In this regard the folded layout, illustrated in Figure 1, is superior to more traditional configurations since the maximum difference in local pavement temperatures between the near and far ends, points A and B, caused by the warm fluid, amounts to only half the difference between inlet and outlet fluid temperature. Furthermore, if the fluid temperature decreases linearly along the pipe, then the average temperature of neighbouring supply and return pipes is constant throughout the system. For these reasons the folded serpentine configuration is capable of providing the most uniform horizontal temperature distribution at the pavement surface.

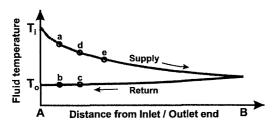


Figure 2:Fluid temperature in folded serpentine

Supply (warm)
Return (cool)
To

Figure1: Folded serpentine piping configuration

Piping layouts for pavement snow melting systems are usually designed assuming that the fluid temperature decreases at a constant rate between supply and return ends of the pipe. When this is the case the energy production rate can be calculated trivially from the product of the flow rate and the inlet-outlet temperature difference. But for long piping runs and low flow rates the assumption of a linear fluid temperature drop is not appropriate. This is illustrated schematically for the folded serpentine in Figure 2 where points **a** to **e** correspond to the positions indicated in Figure 1. In this case it is necessary to calculate the fluid temperature distribution using a suitable model. A complete F.E.M. analysis involves, at the least, two-dimensional

(vertical and transverse) conductive heat flow in the pavement layers coupled with a perpendicular advective flow within the pipes, and it is desirable to develop a simplified approach that can provide results of sufficient accuracy for design purposes with less computational effort.

## 2. Proposed Model:

Substantial mathematical simplification is possible based on simple geometric considerations. The serpentine configuration of Figure 1 can be approximated as a straight pavement panel, Figure 3, subdivided into N elements and N+1 nodes, having a total length A-B and whose cross-section, shown in Figure 4, includes both a supply and a corresponding return pipe. This can be justified on the grounds that the

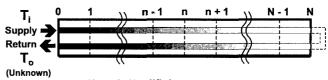


Figure 3: Simplified pavement geometry

difference in fluid temperature (see Figure 2) and, hence, lateral heat flow, is much smaller between neighbouring pipe segments such as **b-c** and **d-e** in Figure 1, than between sections such as **c-d** that include both a supply and a return pipe. Although longitudinal heat flow in the pavement panel (along the direction of the pipes) and underlying soil can be assumed negligible, the resulting simplified geometry, Figure 4, still involves two-dimensional conduction in planes perpendicular to the pipes at longitudinal positions, **n**, along the pipes, coupled with advective flow in both the supply and return pipes, while the fluid temperature is only know beforehand at the inlet end of the system.

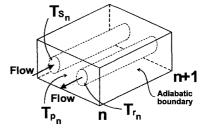


Figure 4: Folded serpentine panel element

The proposed model further simplifies the calculations as follows: The two-dimensional lateral and vertical conductive heat flow in the pavement and soil is replaced by an equivalent one-dimensional vertical heat flow problem to be solved at each longitudinal position,  $\mathbf{n}$ . This is achieved by treating the supply and return pipes as if together they constituted a single point heat source located at the centre line of the pipes. The heat source strength for both supply and return pipes is calculated using an effective heat transfer coefficient  $\alpha$  relating fluid temperature and ambient pavement temperature at the level of the pipes:

$$Qs_n = \alpha A(Ts_n - Tp_n)$$
 and  $Qr_n = \alpha A(Tr_n - Tp_n)$  (1)

where Q is source strength, A is the surface area of the pipe, and  $Ts_n$ ,  $Tr_n$  and  $Tp_n$  are, respectively, the supply and return fluid temperatures and pavement temperature at the corresponding position n. Lateral heat flow through the element walls, Figure 4, is assumed to be negligible since pipes from neighbouring elements are at relatively similar temperatures. These source terms are added to the right hand side of the conduction equation for the vertical pavement and soil temperature profile and, to simplify computation, the source terms are evaluated using temperatures from the "known" time level m.

The temperature at different longitudinal positions within the fluid is calculated from the convective-diffusion equation applied to a cylindrical fluid element, which leads to the upwind difference equation:

$$\rho C_{t} \left( T s_{n}^{m+1} - T s_{n}^{m} \right) / \Delta t = \rho C_{t} q \left( T s_{n-1}^{m+1} - T s_{n}^{m+1} \right) + \alpha A \left( T p_{n}^{m+1} - T s_{n}^{m+1} \right) + \text{fluid conduction}$$
 (2)

where  $\rho C_f$  is the heat capacity of the fluid, q is fluid flow rate, and the remaining terms are as defined previously. An additional equation of similar form is used for the return flow.

With these simplifications, the problem is reduced to the simultaneous solution of two one-dimensional equations for the fluid, coupled with N+1 one-dimensional equations for the pavement-soil temperatures.

### 3. Results:

To verify the simplified model, results were compared with data from a snow melting system incorporated in a concrete garage roof in Fukui City in 1998. Several days data, including vertical temperature profiles in the concrete roof slab, fluid flow rate, inlet and outlet fluid temperatures, and basic meteorological data were measured at 30 minute intervals, during, and in the absence of snow events.

A reasonable test of the validity of the model is its ability to predict fluid outlet temperatures. Figure 5 compares simulated outlet temperatures with those measured in the field. The simulation used handbook derived fluid thermal properties while the thermal conductivity of the concrete was measured using a portable transient plate type apparatus. To eliminate most remaining ambiguities, the simulation model was driven using temperatures measured at points above and below pipe level in the roof slab along with fluid inlet temperatures so that deviations from correct outlet temperature values reflect either deficiencies in the model or inappropriate choices for  $\alpha$ . Figure 5 shows the results obtained using a best choice,  $\alpha$ =60W/m². The discrepancy with measured values is generally much less than 0.1°C, although there is a tendency to briefly underestimate the peak swings by about 0.25°C.

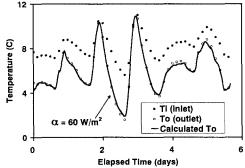


Figure 5: Model verification based on outlet temperature

# 4. Discussion:

Although this simple model provides satisfactory results with minimal calculation effort, several refinements can be envisioned. A more complex heat source formulation involving interaction with other points in the pavement as well as between the supply and return pipes may be potentially useful. Comparison with a more complete F.E.M. model will be carried out to provide the information needed to evaluate the parameter  $\alpha$  for representative pipe-pavement cross sections.

# 5. Conclusion:

The fluid temperature distribution in a folded serpentine snow melting pavement heat exchanger can be satisfactorily modelled by a simple approach that replaces the complex geometrical configuration by an equivalent linear panel that includes both supply and return pipes in a single element and that treats their combined effect on pavement temperatures as a point heat source. This makes it possible to reduce the problem to a simple system of two one-dimensional fluid equations to be solved simultaneously with a small number of one dimensional equations for the vertical heat flow in the pavement and ground.