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Fluid dynamics based verification of the law of similitude for liquefied ground flow

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1. INTRODUCTION

It is important to apply the law of similitude in order to discuss model experimental results in conjunction with case studies on actual ground. Hamada *et al.*[1] proposed the law of similitude for liquefied ground flow using experimental results. We numerically verified the law of similitude in the framework of fluid dynamics.

2. ANALYTICAL METHOD

2.1 Modeling of liquefied soil

Bingham model: In this case the liquefied soil is modeled as a Bingham fluid. Figure 1 shows behavior of liquefied soil by this model [2]. The shear stress τ and shear strain rate $\dot{\gamma}$ relation of the Bingham model is given as:

$$\tau = \eta\dot{\gamma} + \tau_r \qquad (1)$$

where η is the viscosity after yield and τ_r is the yield strength. Uzuoka *et al.*[2] applied the Bingham model to the behavior of liquefied soil.

Pseudoplastic model: Hamada *et al.*[1] studied the mechanisms of liquefaction-induced ground displacements based on, among other techniques, experiments conducted on liquefied ground models. They treated liquefied soil as a pseudoplastic fluid during ground flow. The shear stress shear strain rate relation for such fluids can be expressed using the curve in Figure 2 as follows:

$$\tau = \eta_0\dot{\gamma}/(1 + \dot{\gamma}/\dot{\gamma}_r) \qquad (2)$$

where η_0 is the initial viscosity; $\dot{\gamma}_r$ the shear strain rate when secant viscosity is $\eta_0/2$, called the reference shear strain rate.

2.2 Numerical scheme

The Navier-Stokes equations are used as the governing equations in the flow analysis. The numerical code in our analysis uses the SIMPLE method[3]. The SIMPLE formulation is based on a finite volume discretisation on a staggered grid of the governing equations. For free surface treatment, the VOF method[4] is used.

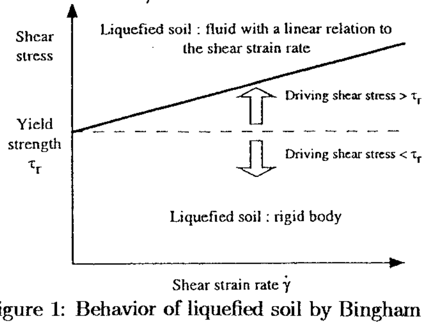

Figure 1: Behavior of liquefied soil by Bingham model

2.3 Law of similitude

In the flow of a liquefied soil, viscous and inertial forces are the most predominant forces. The Reynolds law of similitude is, therefore, applicable here. Assuming the Reynolds number of the actual ground to be equal to that of the model ground, the following equation holds:

$$Re = \rho_m V_m L_m/\eta_m = \rho_p V_p L_p/\eta_p \qquad (3)$$

where ρ is the fluid density; V the velocity of flow; L the characteristic dimension of length; η the fluid viscosity; and suffixes p and m are for actual and model ground, respectively. Putting liquefied ground thickness H as the characteristic linear dimension L, since $\rho_m = \rho_p$ if sand is used as the model ground material, Eqn (3) can be expressed as follows:

$$V_m H_m/\eta_m = V_p H_p/\eta_p \qquad (4)$$

Let the geometrical scale be $\lambda = H_m/H_p$, then we get:

$$\lambda \cdot V_m/V_p = \eta_m/\eta_p \qquad (5)$$

In the pseudoplastic fluid model, the apparent viscosity coefficients for that of the model and actual grounds, respectively, can be expressed as follows:

$$\eta_m = \eta_{0,m}/(1 + \dot{\gamma}/\dot{\gamma}_{r,m}) \qquad \eta_p = \eta_{0,p}/(1 + \dot{\gamma}/\dot{\gamma}_{r,p}) \qquad (6)$$

Since γ and γ_r are non-dimensional numbers, $\gamma_m = \gamma_r$ and $\gamma_{r,m} = \gamma_{r,p}$. Representing the dimension of time with T we have:

$$(\eta_m/\eta_p) = (\eta_{0,m}\dot{\gamma}_{r,m}/\eta_{0,p}\dot{\gamma}_{r,p}) \cdot (T_m/T_p) \qquad (7)$$

In Eqn (7), $\eta_0 \cdot \dot{\gamma}_r$ denotes the shear strength of liquefied soil and is assumed to be in proportion to the vertical confined pressure, i.e. ground layer thickness[1]. Therefore, $(\eta_{0,m}\dot{\gamma}_{r,m})/(\eta_{0,p}\dot{\gamma}_{r,p}) = \lambda$. Hence:

$$(\eta_m/\eta_p) = \lambda \cdot (T_m/T_p) \qquad (8)$$

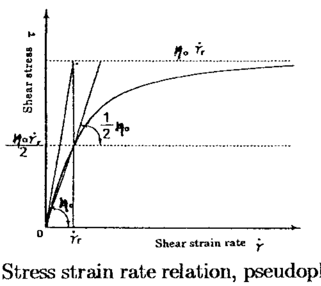

Figure 2: Stress strain rate relation, pseudoplastic fluid

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Substituting Eqn (8) into Eqn (4) gives:

$$V_m = \sqrt{\lambda} V_p \ , \ T_m = \sqrt{\lambda} T_p \tag{9}$$

We can also obtain a law of similitude using the Bingham model. The equivalent viscosity η' for that of the model and actual grounds, respectively, are given as[2]:

$$\eta'_m = \eta_m + (R_r \cdot P_m / \dot{\gamma}_m) \qquad \eta'_p = \eta_p + (R_r \cdot P_p / \dot{\gamma}_p) \tag{10}$$

where R_r is the residual strength ratio. The Bingham viscosity coefficients η_m and η_p are very small (η_m, η_p $\simeq 0$) and can be neglected. Representing the dimension of time with T we get:

$$(\eta'_m / \eta'_p) = (P_m / P_p) \cdot (T_m / T_p) \tag{11}$$

Assuming the pressure P of liquefied soil is in proportion to the ground layer thickness, we have:

$$(\eta'_m / \eta'_p) = \lambda \cdot (T_m / T_p) \tag{12}$$

Substituting Eqn (12) into Eqn (4) gives:

$$V_m = \sqrt{\lambda} V_p \ , \ T_m = \sqrt{\lambda} T_p \tag{13}$$

Therefore, the law of similitude remains the same for both Bingham and pseudoplastic fluid models, i.e. the liquefied ground flow velocity is proportional to the square root of liquefied layer thickness.

3. SIMULATION

The existing non-Newtonian fluid analysis code STREAM is modified to treat the Bingham and pseudoplastic models. Figure 3 shows a two-dimensional numerical model used in the analysis. Different models with different liquefied layer thicknesses were used in the simulations. Table 1 and Table 2 summarize the data used in the analyses. Figures 4 to 7 show the results obtained from the simulation in comparison with the experimental results for 3% and 5% ground slopes.

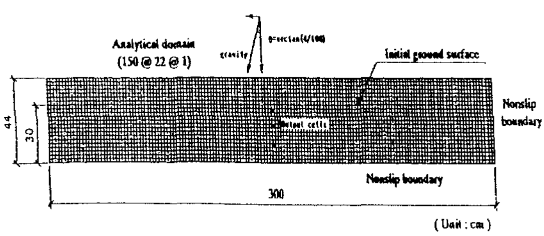

Figure 3: Two-dimensional model used in the analysis

4. CONCLUSIONS

The simulation results reproduced the experimental results very well. It can be clearly seen that the liquefied ground flow velocity is proportional to the square root of liquefied layer thickness. Therefore, the law of similitude holds.

References

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Table 1: data used for the pseudoplastic model

Liquefied soil parameter	Adopted value
Initial viscosity coefficient, η_0	588.6 Pa-s
Reference shear strain rate, $\dot{\gamma}_r$	0.2 1/s
Maximum shear stress ratio	0.0219

Table 2: data used for the Bingham model

η (Pa· s)	R_r	Notes
1.0	0.018	Bingham fluid

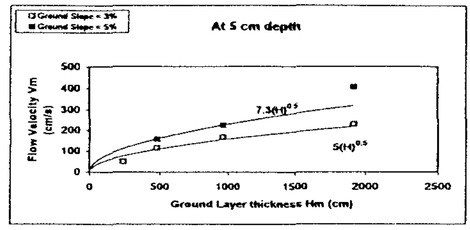

Figure 4: Layer thickness to flow velocity relations, pseudoplastic model.

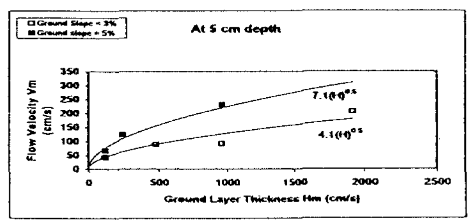

Figure 5: Layer thickness to flow velocity relations, Bingham model.

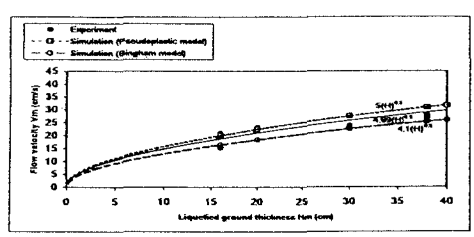

Figure 6: Layer thickness to flow velocity relations for 3% surface gradient.

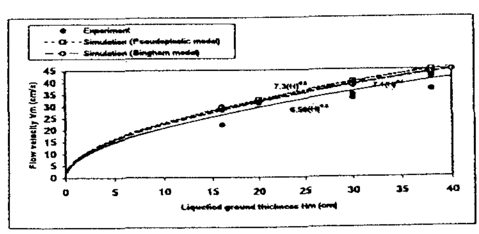

Figure 7: Layer thickness to flow velocity relations for 5% surface gradient.