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Elastic moduli of ice-sand mixture

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INTRODUCTION

Perennially frozen ground can be found in different places around the world which is usually known as permafrost. Various types of permafrost exist due to the range of climatic and geographical conditions. Their structure is a matrix of mineral particles with different size, shape, pore ice, unfrozen water and air voids. The mechanical behaviour is dependent on the interaction of these components, which is strongly influenced by the existing temperature. In this study, the modelling of elastic moduli for an ice-sand mixture as a composite geomaterial is first introduced based on the previous studies presented by Voigt (1889), Reuss (1929) and Omine et al. (1993). The ice-sand mixture is treated as a saturated frozen sand. The applicability of the modelling of elastic moduli for the ice-sand mixture under constant temparature are then discussed from the theoretical and experimental points of view.

ELASTIC MODULI OF AN ICE-SAND MIXTURE WITH ISOTROPIC COMPOSITE MATERIALS

A two phase mixture consists of a basic and a supplementary material. These are called a matrix and an inclusion respectively (see Fig. 1). The ice is herein treated as the matrix and the sand particles are treated as an inclusion. In order to clarify the elastic behaviour of the mixtures, it is necessary to evaluate stress and strain distribution in the mixture. A parameter for estimating the stress distribution in the mixtures has been introduced by Omine, et al. (1993), so that:

$$b = \frac{\bar{\sigma}_s}{\bar{\sigma}_*} \quad (1)$$

where, $\bar{\sigma}_s$ and $\bar{\sigma}_*$ are average stresses applied to the sand inclusion and matrix respectively. Young's modulus and Poisson's ratio of the two-phase mixtures containing isotropic elastic materials were derived using the stress distribution parameter "b" as follows:

$$E = \frac{(b-1)f_s + 1}{\frac{f_s b}{E_s} + (1-f_s)\frac{1}{E_*}} \quad (2), \qquad \nu = \frac{f_s b\frac{\nu_s}{E_s} + (1-f_s)\frac{\nu_*}{E_*}}{\frac{f_s b}{E_s} + (1-f_s)\frac{1}{E_*}} \quad (3),$$

$$b = \left(\frac{E_s}{E_*}\right)^{1/2} \quad (4a), \qquad f_s = \frac{V_s}{V} = \frac{1}{1+e} \quad (4b)$$

where, E_s, E_* and ν_s and ν_* are the Young's moduli and Poisson's ratios of inclusion and matrix respectively, and f_s is volume content of inclusion, with V_s, V and e decribing the volume of sand, total volume and void ratio respectively. The key assumption for representing the stress distribution parameter as a function of the elastic moduli is that the strain energy increments of inclusion and matrix per unit volume are equal so that $dW_s = dW_*$. The detail of this derivation process has already been reported by Omine et al. (1993). In addition, it would be important to point out that when assuming the mixture elements subjected to a uniform strain, parameter b becomes $(E_s/E_*)^1$, and also when assuming the mixtures subjected to a uniform stress, parameter b would be derived as $(E_s/E_*)^0 = 1$. These are known as the Voigt and Reuss models, respectively.

PREDICTION OF ELASTIC MODULI

The Young's modulus was calculated from experimental data by Andersen et al.(1994) on frozen Manchester fine sand at temperatures ranging from −10 to −25 ℃. As a first attempt, the applicability of Eq. (2) is investigated through the comparison of the predicted results with the experimental data, which was obtained from triaxial specimens with special small strain measurement devices. Young' modulus for the sand particles of 90 GPa, which is an average value for quartzite (Lambe and Whitman, 1969), and 9 GPa for polycrystalline ice from Sinha (1989) are used to predict the Young's modulus for the frozen sand. Fig. 2 shows the comparison of experimental data with the predicted results based on the three different models. The results in the range of f_s-values from 0 to 0.7 are shown in Fig. 2. It can be seen that although the experimental data is quite scattered, most of the datapoints plot in-between the results predicted by the Omines and Reuss models. Based on the model used, the Young's modulus increases monotonously but nonlinearly with the increasing volume fraction of sand. The predicted and measured Poisson's ratios are shown in Fig. 3. The measured values are quoted from Baker and Kurfurst (1985), whose data is obtained by an acoustic wave propagation technique. The experimental data is in good agreement with the results predicted by Omines model, which shows that the Poisson's ratios of this frozen sand gradually decreases with the increasing volume fraction of sands.

EFFECT OF SAND PARTICLE STIFFNESS

In basing the ice-sand mixture model, the following analytical results have been obtained (Yasufuku and Springman): 1) The larger the E_s/E_*-value is, the rate of increment of E/E_* with

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f_s-value becomes larger. 2) When the v_s/v_* and f_s-value are the same, the normalized Poisson's ratio, v/v_* increases with increasing E_s/E_*-value. Considering such results, it is important to establish the expected ranges of Young's modulus and Poisson's ratio of the realistic frozen sands. Table 1 has been prepared, based on data reported by Lambe and Whitman (1968). The measured Poisson's ratio v_s and v_s/v_*, and the Young's modulus E_s and E_s/E_* for various original materials are summarized in this table. The realistic range of the Poisson's ratio and Young's modulus of natural sand particles are from 0.12-0.30 and 66-123 GPa, respectively. Figs. 4(a) and (b) show the possible values of the Young's modulus and Poisson's ratio of the ice-sand mixture in the range of f_s from 0 to 0.7 calculated by Omine's model using data from table.1. The differences of both the Young's modulus and the Poisson's ratio of the ice-sand mixture for the geotechnical materials are not so significant, considering the scatter of experimental data shown in Fig.2.

CONCLUSIONS

1) Three different types of models based on the theory of mixtures were introduced to evaluate the elastic moduli for the saturated frozen sands. Although the experimental data is quite scattered, the Omines model gives a good agreement with most of the datapoints. 2) The ranges of expected Elastic moduli of the ice-sand mixtures were investigated using the various geotechnical inclusion materials.

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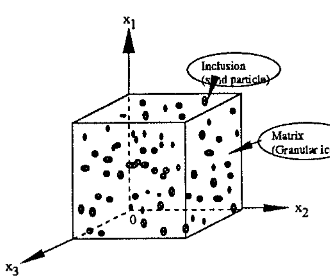

Fig.1 Schematic view of a two phase mixture

Table 1 Poisson's ratio and Young modulus for various materials as inclusion

Material	Poisson's ratio v_s	v_s/v_*	Young's modulus E_s (GPa)	E_s/E_*
Dolomite	0.3	0.97	112-123 (118)	12.4-13.7 (13.1)
Felspathic Gneiss	0.15-0.20 (0.175)	0.48-0.65 (0.56)	84-120 (102)	9.3-13.4 (11.3)
Granite	0.23-0.27 (0.25)	0.74-0.87 (0.81)	74-88 (81)	8.2-9.7 (9.0)
Limestone	0.27-0.30 (0.285)	0.87-0.97 (0.92)	88-109 (99)	9.8-12.1 (11.0)
Mica Schist	0.15-0.20 (0.175)	0.48-0.65 (0.56)	81-103 (92)	8.9-11.4 (10.2)
Obsidian	0.12-0.18 (0.15)	0.39-0.58 (0.48)	66-81 (74)	7.3-9.0 (8.2)
Quartzite	0.12-0.15 (0.135)	0.39-0.48 (0.44)	83-98 (91)	9.3-10.9 (10.1)

* Poisson's ratio and Young modulus for ice: v_*=0.31, E_*=9 GPa (Sinha, 1989)

**:The values in parenthesis indicate the mean values of modulus.

***: This table was made based on table 12.5 summalized by Lambe and Whitman (1968)

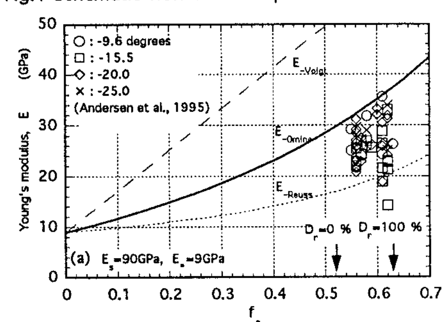

Fig.2 Prediction of Young's modulus by three different model

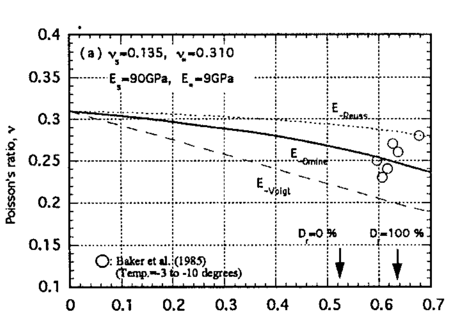

Fig.3 Prediction of Poisson's ratio by three different model

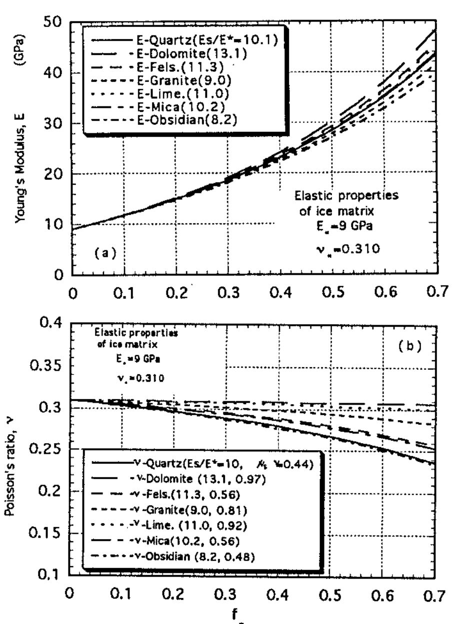

Fig. 4 Predicted elastic properties of ice-sand mixture with differentsand particles as inclusions