III - A423 Critical loads of pile foundation considering the effect of surrounding soil

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I. Introduction

Based on the theory of structural stability, the relationship between the critical load, P_{cr}, of a pile and the length of the pile, l, considering the surrounding soil was discussed in this paper, and it was found that the critical load is $\lim_{l\to\infty} P_{cr}(l) = 2\sqrt{EIk\cos\theta_0}$, in which, EI is the flexural rigidity of the pile, k is the soil spring coefficient and θ_0 is the inclination of the pile. The mathematical calculation also revealed that the effect of the reduction of the soil spring due to liquefaction on the pile foundation was significant and the critical load of a pile in a liquefied layer was very lower than that in nonliquefied layer.

II. Governing equation

The linear governing equation of the single pile with initial deflection, w_0, is given by

$$EIw^{(4)} + Pw'' + k\cos\theta_0 w = -Pw_0'' \qquad w(0) = w''(0) = w(l) = w''(l) = 0 \tag{1}$$

in which, we assume that the both sides of the pile are simply-supported (S.S), w is the deflection of the pile, l is the length, and P is the axial force of the pile(see fig.1 and2).

The dimensionless boundary value problem is written from eq. (1) as

$$W^{(4)} + \lambda W'' + \alpha\cos\theta_0 W = -\lambda W_0'' \qquad W(0) = W''(0) = W(1) = W''(1) = 0 \tag{2}$$

where, $t = \frac{s}{l}$, $W = \frac{w}{l}$, $\lambda = \frac{Pl^2}{EI}$, $\alpha = \frac{kl^4}{EI}$ are dimensionless factors.

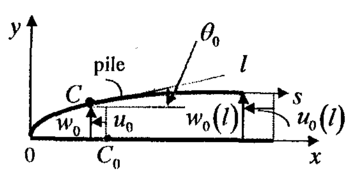

Fig. 1 Initial deflection and coordinate system

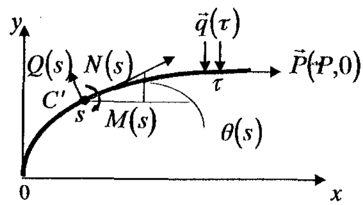

Fig. 2 Notations of pile after deformation caused by external forces

The problems (1) and (2) are valid under the condition that the initial deflection may be large, but the additional deformations due to external force are small, i.e., $w(s)$, $\theta(s) - \theta_0(s)$ are small.

III. Critical load of a pile

Let $\alpha_\theta = \alpha\cos\theta_0$, then the characteristic values of (2) are given as $\lambda_m = m^2\pi^2 + \frac{\alpha_\theta}{m^2\pi^2}$, $(m = 1,2,\ldots)$, and the corresponding characteristic functions are $W_m = \sin m\pi t$. Let $\lambda_{cr} = \min\{\lambda_m\} = \lambda_{m^*}$, then the critical load and the corresponding characteristic function are

$$P_{cr} = \frac{EI}{l^2}\lambda_{cr}, \qquad w_{cr}(s) = \sin\frac{m^*\pi}{l}s \tag{3}$$

Key Words: pile foundation, stability analysis, critical load, nominal hinge

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The minimum value of λ_m is $2\sqrt{\alpha_\theta}$ at the point $m^2\pi^2=\sqrt{\alpha_\theta}$, so we get

$$m^*(l)=m,\quad when \quad m(m-1)\pi^2<\sqrt{\alpha_\theta}<m(m+1)\pi^2 \tag{4}$$

When $k=0$, $\alpha_\theta=0$, then, $m^*=1$, $\lambda_{cr}=\pi^2$, and $P_{cr}=EI\pi^2/l^2\to 0\ (l\to\infty)$, which is the well-known Euler Critical load. The characteristic function is $w_{cr}(s)=\sin\frac{\pi}{l}s$, and it has only one maximum value.

When $k>0$, $\alpha_\theta>0$, m^* in eq. (3) changes corresponding to l, that is $m^*=m^*(l)$. If l is enough long, we can assume that $m^*\geq 2$ and the $[0,l]$ is divided into m^*. The locations when the deflection equals to zero are $s_i=\frac{i}{m^*}l$, $(i=0,1,\dots m^*)$, that is $w_{cr}(s_i)=0$ and $w''_{cr}(s_i)=0$, which correspond to the nominal hinges (see Fig. 3). It means that $P_{cr}(l)$ is the critical load of the following problem in which the length of pile is $\frac{l}{m^*}$

$$EIw^{(4)}+Pw''+k\cos\theta_0 w=-Pw''_0 \qquad w(0)=w''(0)=w(s_1)=w''(s_1)=0 \tag{5}$$

We can calculate the critical load $P_{cr}(l)$ by using eq. (5). From eq. (4) we have

$$\sqrt{1-\frac{1}{m^*}}\cdot\sqrt[4]{\frac{EI}{k\cos\theta_0}}\pi\leq\frac{l}{m^*}\leq\sqrt{1+\frac{1}{m^*}}\cdot\sqrt[4]{\frac{EI}{k\cos\theta_0}}\pi \tag{6}$$

Furthermore, we can get

$$\lim_{l\to\infty}P_{cr}(l)=2\sqrt{EIk\cos\theta_0} \tag{7}$$

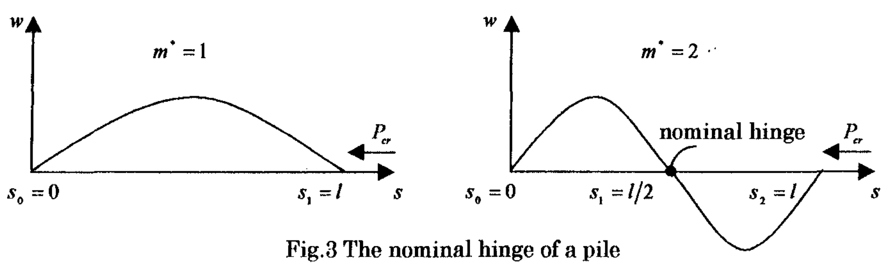

Fig.3 The nominal hinge of a pile

IV. Example

Based on eq. (2), we performed parametric study to obtain the relationship between P_{cr} and l as a function of soil spring, in which $w_0(0)=0$, $\theta_0(s)=const$. First, we examined the influence of N-value on P_{cr}. We analyzed a steel pile with a diameter of 177.8mm, and the soil spring was calculated based on the design code for road bridge in Japan.

Figure 4 shows the result from the analyses. From this figure we can see that P_{cr} are not so strongly dependent on the length but on the N-value when the pile is surrounded by soil.

Figure 5 shows the result from the analyses in which the soil springs are assumed to be reduced due to liquefaction. This figure shows that the effect of the reduction of the soil spring is significant on P_{cr}.

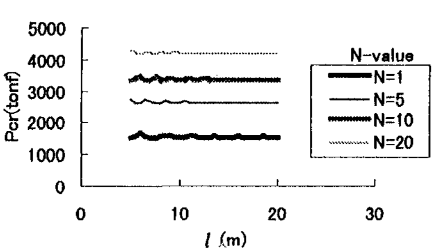

Fig .4 The relationship between l and P_{cr}

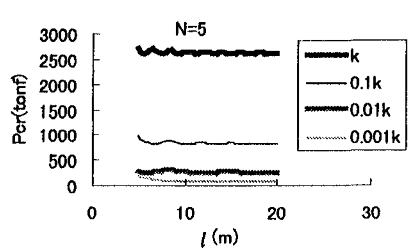

Fig.5 The effect of reduction of soil spring