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## A PARALLEL COMPUTATION OF THE DYNAMIC RESPONSE OF POROUS MATERIAL

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### 1. INTRODUCTION

Generally, soils are complex materials consisting of a granular solid saturated with water or a mixture of air and water. Considering pore flow development of the pore water pressure may influence the dynamic interaction between soils and structures significantly. Hence, idealization of the soils as two-phase coupled solid-fluid material in a FEM system offers more realistic results than an equivalent single-phase homogeneous solid model. However, this approach will require a large-storage, significant running time and high cost. Unless the model is not divided into several subdomains, the memory size for the finite elements very often exceeds the available computer capacity. If we may assign these computations on different processors in a parallel computer machine, the problem is solved and also a solution is obtained at reasonable cost.

In this paper, a series of dynamic analysis of porous material by using a parallel computation is presented. A Finite Element Tearing and Interconnecting (FETI) which is introduced by Farhat<sup>1)</sup> was adopted for parallel computing. Finally, an experimental result based on work of Kawamura<sup>2)</sup> was used to validate this numerical analysis.

### 2. A FEM FORMULATIONS

The equation of motion of porous material<sup>3)</sup> are given as follow,

$$\tau_{ij,j} = \rho \ddot{u}_i + \rho_f \ddot{w}_i \quad (1)$$

$$\pi_{,i} = \rho_f \ddot{u}_i + \frac{1}{f} \rho_f \ddot{w}_i + \frac{1}{k} \dot{w}_i \quad (2)$$

in which  $\tau_{ij}$  are the components of the stress tensor for bulk saturated porous solid;  $\pi$  is the fluid pressure;  $\rho$  and  $\rho_f$  are the mass densities of solid and fluid, respectively;  $k$  is the coefficient of permeability;  $f$  is the porosity; and  $u$  and  $w$  are the displacement field of solid and fluid, respectively. The superposed dot implies time derivative.

Here, the granular solid and the fluid are treated as independent materials with separate material properties with constitutive relations

$$\tau_{ij} = C_{ijkl} \epsilon_{kl} + \alpha m \delta_{ij} (\alpha \delta_{kl} \epsilon_{kl} + \zeta) \quad (3)$$

**Key Words:** parallel computation, porous material, dynamic response, saturated soil layer

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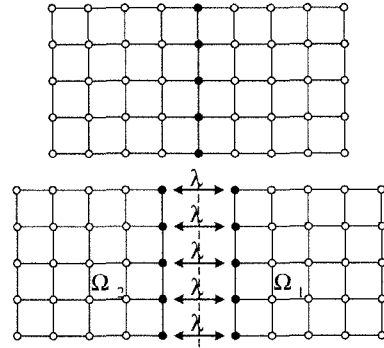


Fig. 1 An Example of Partition of A Domain

$$\pi = m(\alpha \delta_{ij} \epsilon_{ij} + \zeta) \quad (4)$$

where  $\delta_{ij}$  is the Kronecker's delta;  $C_{ijkl}$  are the components of the elasticity tensor;  $m$  is the bulk modulus of the fluid;  $\alpha$  is a measure of compressibility of solid particles representing the contact areas of the particles,  $\alpha$  has a value between 1 and  $f$ ;  $\epsilon_{ij}$  are the components of the solid strain; and  $\zeta$  is the volume change of the fluid.

By applying standard Galerkin Method<sup>4)</sup>, the field of equations of two-phase solid-fluid above are written as semi-discrete equation of motion in a usual manner

$$M\ddot{U}(t) + D\dot{U}(t) + KU(t) = P(t) \quad (5)$$

The detail of development of the foregoing equations were reported in the paper by Ghaboussi<sup>5)</sup>.

The most widely used family of direct time integrating methods for solving (5) is the Newmark's family<sup>6)</sup>. For each computational step, the Newmark method require the solution of a linear algebraic system of equations such as

$$K^* U_{n+1} = P_{n+1}^* \quad (6)$$

### 3. FETI ALGORITHMS

The FETI algorithms were implemented by partition of a overlapping subdomain into a set of totally non-overlapping subdomains, where  $K^e u^e = f^e$ . For example a domain is partitioned into two of subdomains in Figure 1. This governing equilibrium equation is derived by involving the stationary of energy function  $\Pi = \frac{1}{2} u^{eT} K^e u^e - f^{eT} u^e$  subjected to the constraints  $\sum_{j=1}^{N_s} B^{(j)} u^{(j)} = 0$ . The problem is transformed into an unconstrained optimization problem

of the Lagrangian functional

$$L(u, \lambda) = \frac{1}{2} u^{eT} K^e u^e - f^{eT} u^e - \lambda^T \sum_{j=1}^{N_s} B^{(j)} u^{(j)} \quad (7)$$

where  $K^e$  is the block diagonal unassemble global matrix,  $B^{(j)}$  is a signed Boolean matrix which localized a subdomain quantity to the subdomain interface and the Lagrangian multiplier  $\lambda$  represents the interaction forces between the subdomains along their common boundary. Solving the functional with the necessary conditions for  $N_s$  subdomains leads to dual interface problem

$$F_I \lambda = d \quad (8)$$

where

$$F_I = \sum_{s=1}^{N_s} B^{(s)} K^{(s)-1} B^{(s)T}$$

$$d = \sum_{s=1}^{N_s} B^{(s)} K^{(s)-1} f^{(s)}$$

Since  $F_I$  is symmetric positive definite matrix, the iterative solver Conjugate Gradient algorithm is most suitable for computing the unique solution to the unconstraint problem in the parallel computer machine. A preconditioner is needed to enhance its convergence rate. After  $\lambda$  has been defined, the displacements of each subdomain can be evaluated by backward/forward substitution of

$$u^{(s)} = K^{(s)-1} (f^{(s)} - B^{(s)} \lambda) \quad (9)$$

Comparing equation (6) and (8), it is clear that they will have a same form when the dynamic problem is partitioned into several subdomains. If every subdomain is assigned to an individual processor, all local finite element computations, such as forming and assembling the stiffness matrix, can be performed in parallel. As same as a conventional finite element without interprocessor communication. The interprocessor communication is required for the solution of equation (8).

#### 4. A NUMERICAL EXAMPLE

To demonstrate the capability of this parallel computational method, the experimental model based on work of Kawamura<sup>2)</sup> has been chosen. The model was made of saturated Toyoura fine sand and was shaken

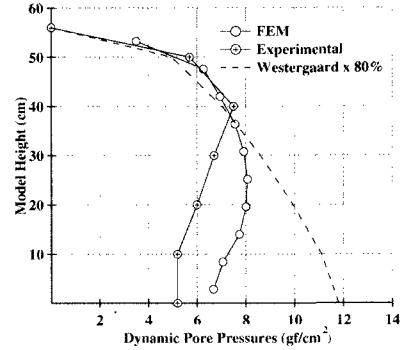


Fig. 2 A Comparison of Dynamic Pore Pressures

on a shaking table with acceleration 300 gals. The model is 200 cm long, 100 cm wide and 56 cm high. The material properties of model are shown in Table 1. Damping ratio of granular material is assumed 10%.

The finite element model consists of 2640 elements and 3289 nodes with more than 17000 degrees of freedoms and was solved by 8 processors in Cray Origin/2000 super computer. A time step  $\Delta t = 0.01$  second was used and integration parameter of Newmark method were  $\beta = \frac{1}{4}$  and  $\gamma = \frac{1}{2}$ . The responses of pore water pressures were picked up on several nodes in the left side of the model. These responses are plotted and compared with their experimental results in Figure 2.

#### 5. CONCLUSION

A good agreement was obtained when the numerical result is compared with experimental result. Also as we expected, the parallel computation have reduced consuming time as well as storage-computer requirement, since each processor is 'responsible' for each subdomain.

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Table 1 Material Properties of Model

$G$ gf/cm <sup>2</sup>	$\nu$	$\rho$ gf/cm <sup>3</sup>	$\alpha$	$m$ gf/cm <sup>2</sup>	$f$	$k$ cm/sec
1.5e5	0.33	1.61	1.0	2.1e7	0.392	0.012