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Contribution Ratio of Tributary Flow to the Peak Discharge of Main Stream

CTI Engineering Co., Ltd., Fellow, Hideaki Kurita[1]
ditto Fumi Takahashi[2]

1. Introduction

In the flood control plan, there are some tributaries that are not included in the design flood discharge of main stream. Additionally, with regard to the tributaries being included in such planning, it has not been clearly verified whether the design discharge of the inflow from tributaries estimated in the main stream plan and that estimated in the tributary plan are truly identical. It is presumed that the aforementioned problems have been caused mainly by the difference in the ratio of tributary discharge to the peak discharge of the main stream at each rainfall (Such ratio is to be referred to as the "flow contribution ratio of discharge" hereinafter). Though it seems on the surface that the problems can be solved by a simple calculation of the catchment area ratio of the tributary to the main stream, as the probability values of the spatial distribution of rainfall on the main stream and tributaries vary to a large degree, no clear calculation has been made to date.

This paper discusses contribution ratio of the tributaries to the peak discharge of the main stream based on the bivariate exponential distribution theory by using 2 variables of tributary discharge and main stream discharge at the peak discharge of main stream as an indicator representing the characteristics of the spatial distribution of rainfall at the confluence.

2. Definition

The case shown in Figure 1 where one tributary flows into a main stream is to be studied. In this paper, the discharge at Point 1 on the main stream before the confluence by the tributary shall be called "the main stream upstream flow". The flow at Point 2 on the tributary shall be called "the tributary flow" and the flow at Point 3 on the main stream after the confluence "the main stream flow. The contribution ratio of the tributary at the reference point 3 at the peak (to be called "the main stream peak flow" hereinafter) shall be defined by using the following formula.

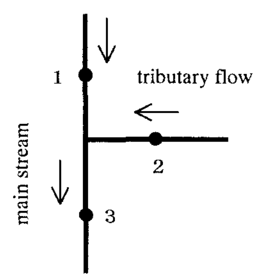

Fig.1 River Channel Model

$$p = \frac{Q_2}{Q_3} = \frac{Q_2}{Q_1 + Q_2} \tag{1}$$

in which Q_1 and Q_2 are the discharge at the time of the main stream peak discharge.

In general, the flow contribution ratio of the tributary (p) is to vary at each rainfall. It may also be presumed that there are the mode and limit value resulting from the geographic characteristics in the catchment area of the main stream and the tributary. Thus, the flow ratio distribution f(p) of Q_2 and Q_3 conforming to the probability distribution must be gained in order to acquire the flow contribution ratio of the tributary to the main stream.

3. Distribution of Contribution Ratio

The main stream upstream flow Q_1 and the tributary flow Q_2 are assumed to conform to the exponential distribution theory. The probability distribution function of the Q_1/Q_2 ratio is as shown in Eq. (1) of the bivariate exponential distribution theory proposed by Nagao et al, by standardizing Q_1 and Q_2.

$$f(\zeta) = (1-\rho)\frac{\zeta + 1}{\{(\zeta+1)^2 - 4\rho\zeta\}^{3/2}} \tag{2}$$

in which ζ is the ratio of the standardized 2 variables $\beta_1 Q_1$ and $\beta_2 Q_2$, thus, $\zeta = \beta_1 Q_1 / \beta_2 Q_2$. $\beta_1$and β_2 are scale parameters. They are $\beta_1 = 1/\overline{Q}_1$ and $\beta_2 = 1/\overline{Q}_2$ in the moment method. ρ is a correlation coeffcient of the 2 variables Q_1 and Q_2, and gained by the following formula. ($\overline{Q}_1$,$\overline{Q}_2$ are average values.)

Authors gained the productivity density function of the flow contribution ratio of the tributary to the peak discharge of the main stream (p) by using the formula (2). This formula of the flow contribution ratio of the tributary (p) shall be interpreted as follows by using β_1 , β_2 .

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[1]Hiroshima Branch Office, CTI Engineering Co., Ltd., 2-31 Hachobori, Naka-ku, Hiroshima 〒730-0013
[2]Osaka Branch Office, CTI Engineering Co., Ltd., 1-2-15 Ohtemae, Chuo-ku, Osaka 〒540-0008

$$p=\frac{Q_1}{Q_1+Q_2}=\frac{1}{1+Q_2/Q_1}=\frac{1}{1+(\beta_1/\beta_2)\xi} \tag{3}$$

in which

$$\xi=g(p)=\frac{\beta_2}{\beta_1}\left(\frac{1}{p}-1\right) \tag{4}$$

Then, Eq. (2) changes as follows by changing variables using the formula (4).

$$f(p)=\frac{d}{dp}\int_1^p f(p)dp=\frac{d\xi}{dp}\frac{d}{d\xi}\int_0^{g(p)} f(\xi)d\xi=\frac{d\xi}{dp}\bullet f\{g(p)\} \tag{5}$$

Finally, the probability density function of the tributary flow contribution ratio to the main stream is as follows.

$$f(p)=-\frac{\beta_2}{\beta_1}\frac{1}{p^2}(1-\rho)\times\frac{\frac{\beta_2}{\beta_1}(\frac{1}{p}-1)+1}{\left[\left\{\frac{\beta_2}{\beta_1}\left(\frac{1}{p}-1\right)+1\right\}^2-4\rho\left\{\frac{\beta_2}{\beta_1}\left(\frac{1}{p}-1\right)\right\}\right]^{3/2}} \tag{6}$$

4. Formula Compatibility on Actual River

Authors have evaluated the compatibility of Eq. (6) on the tributary flow contribution ratio distribution on an actual river. The actual data on 211 floods at the times of the main stream peak flow for the period of 16 years including the flow data on the tributary streams were used to gain the following scale and correlation parameters. They are shown in Figure 1. The probability density function of the tributary flow contribution ratio calculated by applying the above figures to the Eq. (6) and the actual flow contribution frequency in the recorded data are compared in Figure 2.

$$\beta_1=1/\overline{Q_1}=0.00453 \quad \beta_2=1/\overline{Q}_2=0.02350 \tag{7}$$

$$\rho=\overline{Q_1\bullet Q_2}/\overline{Q_1}\bullet\overline{Q_2}-1=0.715 \tag{8}$$

in which Q_2 is the value gained by subtracting Q_1 from Q_3 at each flood.

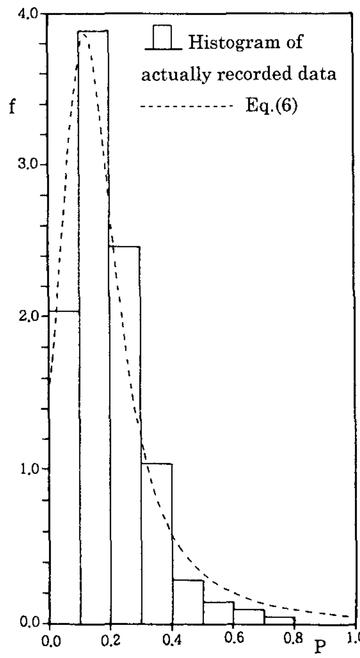

Fig.2 The probability density function of the tributary flow contribution ratio calculated with the formula (6) and the actual flow contribution frequency in the recorded data

5. Conclusion

Eq. (6) was theoretically acquired with no consideration paid to the compatibility with actual rivers. However, the formula has been proven to possess high accuracy in the calculation of the density function of the tributary flow contribution ratio to the main stream and in the actual flood frequency. Though the formula (6) seems a little complex, it can be used even on simple calculators. As conclusion, Eq. (6) verified in this study shall be considered satisfactorily accurate and practical when determining the distribution of the tributary flow contribution ratio at the time of the main stream flow peak.

6. Reference

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