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II -290 Real-time stochastic-dynamic stage and discharge estimation for a channel network

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1 Introduction

Flood routing models based on statistical considerations can improve the forecast of the river stage and discharge by incorporating observations to the numerical model. One of the most efficient ways of developing updating scheme is to employ Kalman filtering. To do so, we need to embed the flood routing model into a stochastic environment by introducing a system noise process in the equations. By using a Kalman filter, the information provided by the stochastic model and the noisy observation data are combined to get an optimal estimate of the state of the system. However, real-time assimilation of data into a numerical model is far from trivial, the computational burden that this technique involves may reduce considerably the interest of the model. Also simplifications have to be done. Our model will use the Reduced Rank Square Root Filter (RRSQRT) which approximates the covariance matrix to avoid high dimensionality.

2 Stochastic dynamic-wave equations

The dynamic-wave flood routing model is based on the Saint Venant equations of unsteady flow. We consider that the inaccuracies of the model are coming mainly from the boundary conditions and the lateral inflows. Therefore, noise terms are added in the continuity equations and in the boundary conditions only.

The spatial dependence of the noise is treated by considering one noise term for every element of the channel network grid.

The discretization of the previous continuous system of equations is made by using the Preissman four-point implicit finite-difference approximation, cf Cunge [2].

3 Noise process

We may consider the noise as white Gaussian process, but it appears that such a process, in many cases, does not represent the reality, cf Takasao and Shiiba [4]. Therefore we will consider a colored noise process.

On the opposite of the white Gaussian noise processes, a colored noise process depends on its previous values. It implies that we need to propagate the colored noise terms p from one time-step to the following time-step.

$$p(x_i, t_{j+1}) = e^{-\Delta t/\tau} p(x_i, t_j) + v(x, t_j, t_{j+1}) \qquad (1)$$

with t_j being the previous time-step, v is white Gaussian. By using a colored noise process, the system of equations become:

$$F(Y_{j+1}, Y_j) = G_p p_j + G_v v_j \qquad (2)$$

in which G_p and G_v are interpolation matrix and Y is the state variable, composed of the water stage and discharge at every cross-sections in the network.

The state variable is supposed to be Gaussian, also we compute at every time-step its expectation and its covariance matrix.

4 RRSQRT filter

The resolution of the system of equations 2 is performed by the RRSQRT filter. The main idea in this algorithm is to approximate the error covariance matrix P by a matrix of lower rank, cf Heemink [3]. This will be done by decomposing the error covariance matrix in an eigen value decomposition and then to consider only the highest eigen values. To preserve the fact that all the eigen values are positive (P is a semi-positive matrix), we will work on square-root factorization of $P = SS^T$.

The algorithm can be decomposed in three steps.

- Initial truncation: we compute the sorted eigen decomposition: $P(0) = V(0)D(0)V(0)^T$, where $V(0)$ is an orthogonal matrix and $D(0)$ is the diagonal matrix with the eigen values on the diagonal sorted from the highest value. Then the approximate initial

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covariance matrix is obtained by truncating (deleting) all columns right from the column corresponding to the smallest eigen values kept in $D(0)$ and the corresponding column in $V(0)$.

- Propagation: from time $t = t_j$ to time $t = t_{j+1}$, we propagate the expectation of the state variable vector and the square-root covariance matrix. These equations are derived from equation 2 by a Taylor expansion. The propagation step increases the dimension of the square-root covariance matrix. Also this matrix has to be truncated to keep a small dimension.
- Truncation of the square-root ocvariance matrix: the algorithm of this truncation is similar to the one of the initial truncation.
- Filter: at time observations are available, the expectation of the state variable and the square-root covariance matrix are updated. The Potter filter is used in this model, cf Bierman [1].

5 Application The model is applied to a single four-km long channel put into a four dimension noise grid. The upstream boundary condition is a constant discharge, the downstream boundary is a tidal wave. The lateral inflow is a hydrograph, uniform along the channel. The water depth is measured at every 30 minutes at cross-section number 25. The results of this application are given in Figure 1, 2 and 3. they show that the colored noise filter improves the estimation.

6 Conclusions A dynamic-stochastic model was presented. To deal with the difficulties that the Kalman filter involves, the RRSQRT filter was used. The problem of the noise was solved by using colored noise process. An application of this model to a one-channel reach was performed. From this simple example, it appears that the model makes a precise estimation of the state of the channel, without adding too much computational time as compared as the deterministic model. Further researches have to be made on the noise process (use of a multiplicative noise process and the spatial covariance structure). Also the model will be extended to deal with channel networks.

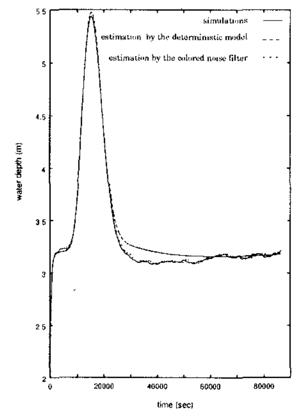

Fig. 1 estimation by the colored noise filter and by the deterministic model, at the cross-section number 25

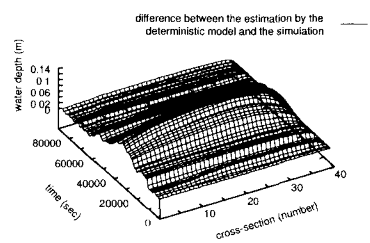

Fig. 2 absolute difference between simulation and deterministic model

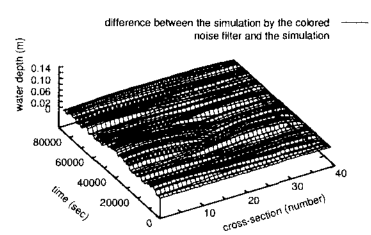

Fig. 3 absolute difference between simulation and colored-filter model

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