II -34

A New Phase-Averaged Model and Simulation of Carrier and Associated Long Wave Evolution

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1. Introduction

Narrow-banded carrier waves have been long recognized as one of the primary driving mechanisms for long period fluctuations. Although there are many phase-resolving type models such as Boussinesq equations (Mizuguchi 1996; Toita & Mizuguchi 1996; Kioka, Yamane & Aoki 1996), generalized nonlinear dispersive wave model by Nadaoka et al. (1994,1997), etc. which can simulate the wave evolution with high performance in numerical simulation, they need relatively long CPU time. But we may assume that the carrier waves are slowly modulated as they propagate in a dispersive medium such as water and group forms can be assumed for carrier waves instead of fluctuating random carrier waves. Therefore it would obviously be a desirable prospect to have a phase-averaged type wave model that can describe both the carrier wave group and accompanying long period wave with reasonable computational efficiency. The recently developed phase-averaged Boussinesq model (Nadaoka and Raveenthiran, 1998) has some discrepancies due to the limitation in the applicable domain of the improved Boussinesq equations. Therefore the work presented here may be viewed as a generalization of the preceding model by employing the generalized nonlinear dispersive wave model by Nadaoka et al. (1994, 1997) as the governing equations and the limitation of the previous phase-averaged Boussinesq model may be removed.

2. Long and carrier wave evolution

Recently Nadaoka et al. (1994,1997) introduced a set of fully dispersive weakly nonlinear wave equations describing wave transformations over varying depth. These equations in their most general form are composed of several depth-dependence functions, each contributing to the dispersivity of the full set. Therefore in the present work these equations are used as the governing equations to derive the new phase averaged-type model because there is no inherit depth restriction on the validity range of these equations. In these equations, the velocity field is represented by a few vertical-dependence functions having different wave-numbers. So the long and carrier wave velocity components may be expressed in similar way such as

$$u = \sum_{m=1}^{N} \left\{ u_{lm}(x,t) + \tfrac{1}{2} A_{um}(x,t) \exp\left[i\left(\int k dx - \omega t \right) \right] + * \right\} \tag{1}$$

in which $u_{lm}(x,t)$ and $A_{um}(x,t)$ are the m^{th} velocity components of long and carrier waves respectively, k represents the wave number to be computed according to the linear dispersion relation for a prescribed dominant frequency ω and a local depth h. The free surface displacement of the wave field may be given as

$$\eta = \eta_l(x,t) + \tfrac{1}{2} A(x,t) \exp\left[i\left(\int k dx - \omega t \right) \right] + * \tag{2}$$

Here carrier waves are assumed to be narrow-banded and $A(x,t)$ and $A_{um}(x,t)$ are slowly varying complex amplitude quantities. For simplicity the subscripts of long wave components will be dropped from here on. By using the method of multiple scales the following equations may be derived.

$$\frac{\partial \eta}{\partial t} + \sum_{m=1}^{N} \frac{\partial}{\partial x}\left[\left(\frac{\omega_m^2}{g k_m^2} + \eta \right) u_m + \tfrac{1}{4}\left(A A_{um}^{*} + A^{*} A_{um} \right) \right] = 0 \tag{3}$$

$$\frac{\partial A}{\partial t} + \sum_{m=1}^{N} \frac{\partial}{\partial x}\left[\left(\frac{\omega_m^2}{g k_m^2} + \eta \right) A_{um} + A u_m \right] - i\omega A + ik \sum_{m=1}^{N} \left[\left(\frac{\omega_m^2}{g k_m^2} + \eta \right) A_{um} + A u_m \right] = 0 \tag{4}$$

$$\sum_{m=1}^{N} A_{nm} \frac{\partial u_m}{\partial t} + g B_n \frac{\partial \eta}{\partial x} + B_n \frac{\partial}{\partial x}(NLT1) = 0 \tag{5}$$

$$\sum_{m=1}^{N} \left(A_{nm} + k^2 C_{nm} - i k_x C_{nm} - ik D_{nm} \right) \frac{\partial A_{um}}{\partial t} + g B_n \frac{\partial A}{\partial x} + B_n \frac{\partial}{\partial x}(NLT2) + ik B_n \left(gA + NLT2 \right) =$$

$$\sum_{m=1}^{N} \left[\omega\left(2k C_{nm} - i D_{nm} \right) \frac{\partial A_{um}}{\partial x} + 2ik C_{nm} \frac{\partial^2 A_{um}}{\partial t \partial x} - i\omega C_{nm} \frac{\partial^2 A_{um}}{\partial x^2} + \omega\left(i A_{nm} + i k^2 C_{nm} + k_x C_{nm} + k D_{nm} \right) A_{um} \right] \tag{6}$$

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Here the parameters A_{nm}, B_n, C_{nm} and D_{nm} are defined in the phase-resolving new wave equations (see Nadaoka et al 1997). $NLT1$ and $NLT2$ are second order nonlinear quantities in terms of u_m and A_{um}. Equations (3) and (5) are the long wave equations while the carrier wave are expressed by equations (4) and (6).

3. Sample simulations

Two cases are presented below as demonstrative simulations: single wave group over uniform depth (Figure 1) and mild-slope bottom topography (Figure 2). The results from these computations are compared with those from the phase-resolving new wave equations stated above. The present model shortens the CPU time considerably compared with the phase-resolving new wave equations. It is 30-times for uniform depth and 23-times for mild-slope bottom topography. Additional tests from very deep to shallow water regions have further confirmed the accuracy of the model, however these comparisons had to be excluded due to space limitations.

(*a*) (*b*)

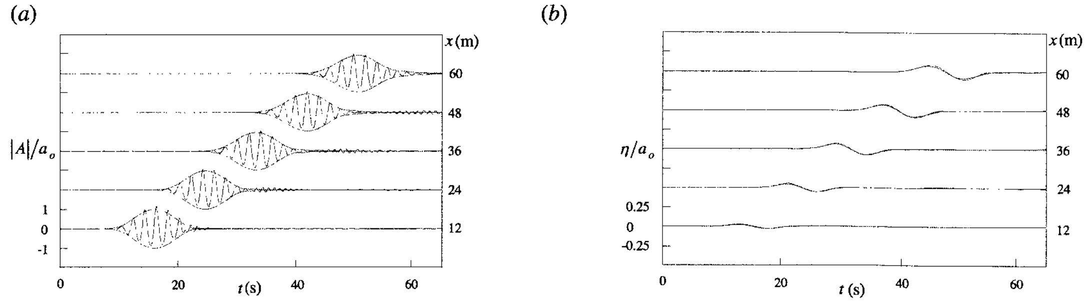

Figure 1. Numerical simulations for uniform depth (h = 25 cm): present model (solid line)compared with the phase-resolving new wave equations (dashed line) (*a*) carrier wave (*b*) long wave (a_o = 1 cm)

(*a*) (*b*)

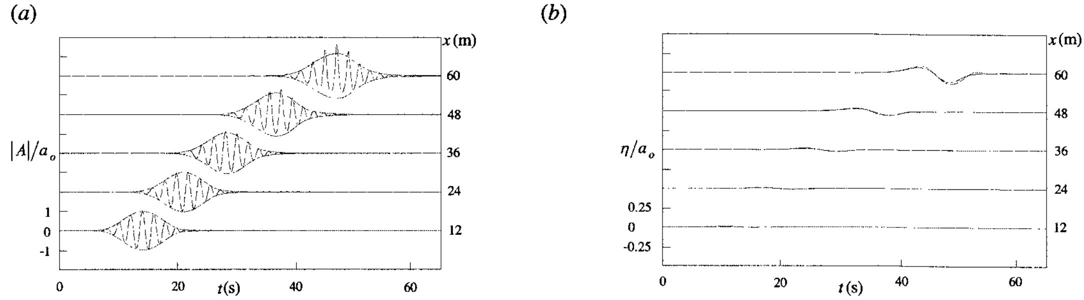

Figure 2. Numerical simulation for mild-slope (1:100): present model (solid line)compared with the phase-resolving new wave equations (dashed line) (*a*) carrier wave (*b*) long wave (a_o = 1 cm)

4. Concluding remarks

A new phase-averaged type water wave model has been derived using the new wave equations, in which the applicable domain (from very deep to shallow water regions) is quite higher than that of the previous phase-averaged Boussinesq model. Numerical examples with comparison to phase-resolving new wave equations show the validity of the present wave model. The performance of the model in CPU time is considerably higher than that of the previous phase-averaged Boussinesq model.

5. References

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