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I −A3　Reliability and Entropy in Structural Design

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1.Outline

The informational entropy which indicates a degree of uncertainty is considered to be helpful to choose a design alternative in structural reliability problems. A question has arisen on a design decision process. Which one should be employed if two alternatives have a same level of reliability, but have noticeably different degrees of uncertainty. This study will answer the problem by demonstrating how useful the concept of entropy is as a measure of information and consequently how comprehensively we can gain the insight into reliability vs. uncertainty. A basic example is numerically demonstrated to discuss the problem and to show the entropy is useful as well as reliability.

2.Introduction

Reliability is a measure to evaluate quantitatively safety of structural systems and our community has accepted this measure as a rational target probability of structural performance in design and construction. In fact, concept of performance based design is greatly indebted to reliability theory with which many probabilistic engineers have been developing the basis. However, a question has arisen to the author on a problem; Which design alternative should be employed if two design alternatives have a same level of reliability, but have noticeably different degrees of uncertainty. Is reliability sufficient to deal with uncertainties and to discuss superiority of design alternatives?

This study will try to answer the problem by demonstrating how useful the concept of entropy (Khinchin 1957, Kullback 1959, Hoshiya and Yamamoto 1998) as a measure of information and how comprehensively we can gain an insight into reliability vs. uncertainty. To help understand this problem, a simple numerical example is used through the study.

3.Reliability and Entropy

Consider a basic reliability problem in which the performance function is given by

$$Z = R - S \tag{1}$$

For simplicity, both R and S are Gaussian random variables and they are mutually independent. The load S is assumed to be Gaussian with the mean 2 and the standard deviation 1.5. Note that in what follows the units of numerical values are purposely dropped because they are not necessary in the discussions. Now compare two design alternatives; design A having the mean 8 and the standard deviation almost 0 of the resistance R, and design B having the mean 12 and the standard deviation 2 of the resistance R.

We set up the design alternatives A and B such that they have a same probability of failure. The central safety factors of μ_R / μ_S and the characteristic safety factors of R^* / S^*, where the probability that S exceeds S^* is 0.05 and the probability that R stays below R^* is 0.05, are also evaluated. Together with the entropy values of the design alternatives A and B based on eq(2), they are tabled in Table 1.

It is noted that performance function Z represents the nature of the design, and the entropy of Z may be defined by

$$H = -\int f(z) \ln f(z) dz \tag{2}$$

where $f(z)$ is the probability density function of Z. The integration is performed over the whole range of Z. It is clear that if the standard deviation increases, the entropy increases and if $f(z)$ is a dirac delter function, that is, if Z is

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deterministic, H becomes zero. Consequently, the greater the entropy is, the more uncertainty involves in the design.

Very interesting observations are made from Table 1.

(1) Designs A and B are equal in term of reliability.

(2) However, if the central safety factor or the characteristic safety factor are taken into account, design B may be the better design.

(3) The entropy of design A is smaller than that of design B. Therefore, design A involves less uncertainty and may be the better design. Note that H_F which is the entropy of the subsystem of failure is same both for designs A and B.

(4) The total costs for designs A and B are more or less same, since design A employs material of excellent quality having less deviation than design B, and consequently the unit price is higher, however the design volume can be smaller than design B. On the contrary, design B has low unit price but requires greater volume because of greater deviation of material.

Table 1　Design Characteristics

	Design A	Design B	Evaluation
P_f	3.17E-05	3.17E-0.5	A=B
central SF	4.0	6.0	A<B
characteristic SF	1.79	1.96	A<B
H	1.8244	2.3352	A>B
H_F	0.0003	0.0003	
H_S	1.8241	2.3349	
total cost	$200	$200	A=B

Finally, the author's conclusion is to employ the design A based on the degree of uncertainty despite of the same reliability.

4.Conclusion

The discussions on an example suggest the potential usefulness of entropy to help evaluate design alternatives. Incorporation of entropy in reliability analysis still remains unsolved and physical interpretation of entropy is necessary to gain insight into the problem.

References

1 Hoshiya, M. and Yamamoto, K., Role of Entropy in Safety Evaluation of Structural Systems, USA-China-Japan Workshop on Civil Infrastructure Systems, Shanghai, China, Nov., 1998

2 Khinchin, A. I., Mathematical Foundations of Information Theory, Dover Publications, New York, 1957

3 Kullback, S., Information Theory and Statistics, Dover Publications, New York, 1959