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Application of the Gray System Theory in Analyzing the Effect of the Particle Size Distribution of Fly Ash on Cement Strength

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1 INTRODUCTION

The used amount of fly ash in cement manufacture is usually restricted due to the well-known drawbacks of fly ash blended cement (FAC for short) such as slow hardening, low early strength and so on. For a given Portland cement clinker and a given addition of fly ash the strength development of FAC concrete is mainly depended on the pozzolanic activity of the used fly ash and the workability of fresh concrete and the microstructure of hardened cement. In this experiment the influence of the particle size distribution of fly ash on its pozzolanic activity and the performances of cement was studied with the gray system theory. By considering the particle size distribution of fly ash as a gray system, the authors established a gray model so as to investigate: (1) the relation between FAC properties and the particle size distribution of fly ash; and (2) what size particles mainly contribute to FAC strength.

2 BASIC PRINCIPLE OF THE GRAY SYSTEM THEORY

Prof. Deng founded the gray system theory in 1980s[1]. A gray system is such a system that one part of its information is known while the other is unknown. As the relations among the various factors contained in a gray system being very complicated relevant degree is especially introduced to quantitatively express the interrelevance among gray factors. According to the relevant degree it can be easily understood that among the various factors which plays a leading role in affecting the properties of the system. Giving a time series $\{X_0(k)\}$, k=1, 2, ..., N, and calling it as a parent series, correspondingly, giving another$\{X_i(k)\}$, i=1, 2, ... M and taking it as a sub-series. At time k the relevant coefficient $\xi 0_i(k)$ between X_i and X_0 can be calculated as [1], in which ρ is a resolution factor, generally ρ = 0.5. Therefore, the relevant degree between X_0 (k) and X_i (k) is given in [2].

Remembering $\sigma_i=\sum_{K=1}^{N} KX_i(k) - \sum_{K=1}^{N} X_i(k) \cdot \sum_{K=1}^{N} K/N$, and $\sigma_k=\sum_{K=1}^{N} K^2-(\sum_{K=1}^{N} K)^2/N$, if $sgn(\sigma_i/\sigma_k) = sgn(\sigma_0/\sigma_k)$, it can be said that X_i is positive relevant with X_j. When $sgn(\sigma_i/\sigma_k) = -sgn(\sigma_0/\sigma_k)$, X_i is negative relevant with X_0.

Taking COLγj as the column j of a relevant matrix, i.e. COLγj=$[\gamma_{1j}, \gamma_{2j}, \gamma_{3j}, ... \gamma_{mj}]$, if COL$\gamma$j* = max COL$\gamma$j, the COL$\gamma$j* is denominated as the strongest relevant column, correspondingly, the X_{j*} will be a factor which will exert the strongest effect on the parent factor. X_{j*}, therefore, is called a key factor in the investigated system.

3 EXPERIMENTS AND RESULTS

3·1 Fly ash used in the experiment

Two kinds of fly ash were used in the experiment were from different electric power plants, one was called FA1, the other was FA2. Their chemical compositions and particle size distributions are shown in **Table 1**. According to China National Standard GB-2847 (pozzolanic admixtures in cement making) the pozzolanic activity of the two fly ashes was examined. The results are shown in **Table 2**.

Table 2 Evaluation of the pozzolanic activity of fly ash

Sample	OPC[1)]	70%OPC +30%FA1	70%OPC +30%FA2
Strength (MPa)[2)]	53.2(100)[3)]	42.9(80.6)	39.6(74.5)

1): ordinary Portland cement; 2): 28d compressive strength; 3): figures in brackets are strength ratio referred to OPC (%)

3·2 Effect of grinding time of fly ash on its powder properties and cement performances

FA2 was ground in a ball mill for 20, 30, 40 and 60 min, respectively. With the prolongation of grinding its median diameter, characteristic diameter and gravity are decreased, and its specific surface area is considerably increased. The variation of its particle size distribution with grinding time is listed in **Table 3**, which indicates the longer the grinding time, the less the large particles in fly ash. With the re-placement of 30% Portland cement with FA2 or ground FA2 mortar specimens of 40×40×160 mm were prepared at w/c=0.46 and s/c=2.50, then cured in water at 20±2℃ for different ages. From **Table 3** it can be found that FAC strength is increased with the grinding time of fly ash. By taking the 7d and 28d compressive strengths as parent series and the content (weight percentage) of fly ash particles with diameters within the given ranges as sub-series (**Table 3**), the relevant degrees and relevant polarities between cement strength and the particle size distribution of fly ash were calculated. The results are shown in **Table 4**.

4 ANALYSES AND DISCUSSIONS

Table 1 shows that FA1 and FA2 have close chemical compositions and are belong to low-calcium fly ash. Generally speaking, the more vitreous material that fly ash contains, the better pozzolanic activity it will have. It seems that FA2 is more active than FA1. But, practically, FA1 possesses better pozzolanic activity than FA2 does (**Table 2**). According to the viewpoints of Dhir(2) and Mehta(3) and the results of this experiment the main reason for this can be attributed to the difference between their particle size distributions (**Table 1**).

From **Table 4** it can be found that the relevant degrees between 7d compressive strength of FAC and the particle size ranges are in the following order: 0.981>0.946>0.945> 0.878>0.730>0.600. This means that the contribution of fly ash to 7d comp. strength of FAC is in below sequence based on its particle size distribution: {10-20μm}>{<10μm}>{20-30μm}>{30-45μm}>{45-80μm}>{>80μm}; for 28d

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$$\xi_{0i}(k) = \frac{\min_i \min_k |X_0(k) - X_i(k)| + \rho \max_i \max_k |X_0(k) - X_i(k)|}{|X_0(k) - X_i(k)| + \rho \max_i \max_k |X_0(k) - X_i(k)|} \quad [1]; \qquad r_{0i}(k) = 1/N \cdot \sum_{k=1}^{N} \xi_{0i}(k) \quad [2]$$

Table 1 Chemical compositions and particle size distributions of the used fly ash

Sample	Ig. L.[1]	SiO_2	Al_2O_3	Fe_2O_3	CaO	MgO	Particle size distribution (%) $D^{2)} \leqq 30\mu m$	$D \geqq 45\mu m$	$D > 80\mu m$	Median D (μm)	Characteristic D (μm)	Spec. surface area (m^2/kg)
FA1[3]	7.85	54.52	24.92	6.55	4.25	1.15	51.2	38.7	11.8	28.4	43.0	465.9
FA2	2.22	56.94	25.34	5.49	3.83	4.64	40.7	45.6	20.1	39.7	55.5	365.9

1) : Ignition loss; 2) : particle diameter; 3) : XRD patterns showed that there was more vitreous material in FA2 than in FA1.

Table 3 Selected parent series and sub-series for the analysis with the gray system theory

k	Grinding Time (min)	Parent series (Strength) $X_{01}(k)$ 7d (MPa)	$X_{02}(k)$ 28d (MPa)	Sub-series (particle size ranges-weight percentage) X1(k) <10μm	X2(k) 10-20μm	X3(k) 20-30μm	X4(k) 30-45μm	X5(k) 45-80μm	X6(k) >80μm
1	0	21.3	37.2	15.27	14.48	10.99	13.69	21.52	24.05
2	20	31.0	46.1	41.77	23.43	15.36	14.27	4.94	0.23
3	30	31.9	50.4	46.96	23.04	13.40	11.78	4.55	0.27
4	40	33.1	53.7	47.02	26.51	16.96	8.65	0.85	0.01
5	60	36.8	55.2	49.51	26.98	13.66	8.47	1.35	0.03

Table 4 Relevant degree and polarity between compressive strengths and particle sizes of fly ash

Particle size ranges	<10μm	10-20μm	20-30μm	30-45μm	45-80μm	>80μm
7d Compressive strength	0.946	0.981	0.945	-0.878[1]	-0.730	-0.600
28d Compressive strength	0.933	0.969	0.954	-0.878	-0.717	-0.600

1): minus figures mean negative relevance

compressive strength of FAC there will be: 0.969>0.954>0.933>0.878>0.717>0.600, and {10-20μm}>{20-30μm}>{<10μm}>{30-45μm}>{45-80μm}>{>80μm}.

By rewriting **Table 4** in a matrix form as follows:

$$\gamma = \begin{bmatrix} 0.946 & 0.981 & 0.945 & 0.878 & 0.713 & 0.600 \\ 0.933 & 0.969 & 0.954 & 0.878 & 0.717 & 0.600 \end{bmatrix} \quad [3]$$

it can be easily found that the content of 10-20μm particles in fly ash is a key factor in controlling both 7d and 28d strength of FAC; and **Table 4** shows that (1) fly ash particles with diameter less than 30μm have relevant degrees(>0.90) with FAC strength; and (2) those with diameter larger than 30 μm are negative relevant to FAC strength. The former reveals that the particles with diameter <30μm have a positive effect on the FAC strength, among which 10-20μm particles, being the key factor, have the greatest contribution to FAC strength; the latter indicates that FAC strength will be decreased with the increase of the content of lager than 30μm particles, especially that of >45μm particles. Therefore, for the improvement of the strength of FAC with a given addition of fly ash it is suggested to increase the content of <30μm particles and reduce that of >45μm particles in fly ash, which can be achieved by pre-grinding fly ash before adding it to cement.

An appropriate time for pre-grinding fly ash is about 20 minutes because after ground for that time the content of <30 μm particles in fly ash has accounted for above 80% and 7d, 28d strengths of FAC have been respectively increased by 45.5% and 23.9% compared with the cement blended with unground fly ash (**Table 3**).

By considering various harmonious factors and rejecting unharmonious factors a gray model GM(1,3) was selected and established for indicating the relationship between 28d FAC compressive strength and the contents of fly ash particles of <30μm (X_1) and >45μm (X_2) in diameter as:

$$dX_1^{(1)}/dt + 0.686X_1^{(1)} = 0.662X_2^{(1)} + 0.170X_3^{(1)} \quad [4]$$

its differential analog equation is as follows:

$$X_1^{(0)}(k) = 0.662X_2^{(1)}(k) + 0.170X_3^{(1)}(k) - 0.686Z_1^{(1)}(k) \quad [5]$$

where, $Z_1^{(1)}(k) = 0.5\,X_1^{(1)}(k) + 0.5X_1^{(1)}(k-1)$.

From equations [4] and [5] it can be directly understood the relationship between FAC strength and the content of fly ash particles with diameter <30μm and >45μm.

5 CONCLUSIONS

Based on this study following conclusions were obtained: (1) the pozzolanic activity of the investigated fly ash has a close bearing on its particle size distribution, and it is possible to predict FAC strength with a given fly ash addition according to the particle size distribution of the added fly ash by establishing a gray model; (2) the content of fly ash particles with diameter of 10-20 μm has the maximal relevant degree with FAC strength, the particles >30μm in diameter have a negative effect on the strength of FAC. In order to improve the strength of FAC it is suggested to increase the content of <30μm particles and reduce that of >45μm particles in fly ash by pre-grinding fly ash before use; and (3) another possible application of the gray system theory in practice is production and quality control.

6 REFERENCES

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