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Analytical solution of consolidation of multi-layered ground with vertical drain

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INTRODUCTION

The closed-form solution of consolidation of double-layered ground with vertical drain has been obtained by Tang (1997). Vertical drains are usually installed in subsoil consisting of several layers. A closed-form solution for multi-layered ground with vertical drain is presented.

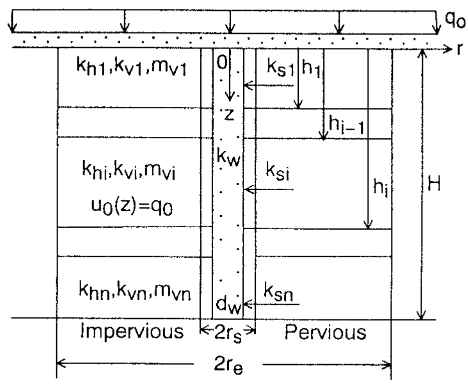

Fig. 1 Analysis scheme

MATHEMATICAL MODELLING

$$\frac{k_{si}}{m_{vi}\gamma_w}\left(\frac{1}{r}\frac{\partial u_{si}}{\partial r}+\frac{\partial^2 u_{si}}{\partial r^2}\right)+\frac{k_{vi}}{m_{vi}\gamma_w}\frac{\partial^2 \bar{u}_i}{\partial z^2}=\frac{\partial \bar{u}_i}{\partial t} \quad (1)$$

$$\frac{k_{hi}}{m_{vi}\gamma_w}\left(\frac{1}{r}\frac{\partial u_{ni}}{\partial r}+\frac{\partial^2 u_{ni}}{\partial r^2}\right)+\frac{k_{vi}}{m_{vi}\gamma_w}\frac{\partial^2 \bar{u}_i}{\partial z^2}=\frac{\partial \bar{u}_i}{\partial t} \quad (2)$$

$$\frac{\partial^2 u_{wi}}{\partial z^2}=-\frac{2}{r_w}\frac{k_{si}}{k_w}\left(\frac{\partial u_{si}}{\partial r}\right)\bigg|_{r=r_w} \qquad i=1,2,\cdots n \quad (3)$$

$$\bar{u}_i=\frac{1}{\pi(r_e^2-r_w^2)}\left(\int_{r_w}^{r_s}2\pi u_{si}dr+\int_{r_s}^{r_e}2\pi u_{ni}dr\right) \quad (4)$$

Continuous conditions: $z=h_i$, $k_{vi-1}\frac{\partial \bar{u}_{i-1}}{\partial z}=k_{vi}\frac{\partial \bar{u}_i}{\partial z}$, $\frac{\partial u_{wi-1}}{\partial z}=\frac{\partial u_{wi}}{\partial z}$, $u_{wi-1}=u_{wi}$, $\bar{u}_{i-1}=\bar{u}_i$

SOLUTION OF SYSTEM

The solutions of $\bar{u}_1$, $\bar{u}_i$ and $\bar{u}_n$ are:

$$\bar{u}_1=\sum_{m=0}^{\infty}A_m g_{m1}(z)e^{-\beta_m t} \quad (5) \qquad \bar{u}_i=\sum_{m=0}^{\infty}A_m g_{mi}(z)e^{-\beta_m t} \quad (6) \qquad \bar{u}_n=\sum_{m=0}^{\infty}A_m g_{mn}(z)e^{-\beta_m t} \quad (7)$$

where:,
$$g_{m1}(z)=\eta_{m1}\sin\left(\lambda_{m1}\frac{z}{H}\right)+c_{m1}\zeta_{m1}\sinh\left(\xi_{m1}\frac{z}{H}\right) \quad (8)$$

$$g_{mi}(z)=a_{mi}\eta_{mi}\sin\left(\lambda_{mi}\frac{z}{H}\right)+b_{mi}\eta_{mi}\cos\left(\lambda_{mi}\frac{z}{H}\right)+c_{mi}\zeta_{mi}\sinh\left(\xi_{mi}\frac{z}{H}\right)+d_{mi}\zeta_{mi}\cosh\left(\xi_{mi}\frac{z}{H}\right) \quad (9)$$

$$g_{mn}(z)=b_{mn}\eta_{mn}\cos\left[\lambda_{mn}\left(1-\frac{z}{H}\right)\right]+d_{mn}\zeta_{mn}\cosh\left[\xi_{mn}\left(1-\frac{z}{H}\right)\right] \quad \text{(for impervious base)} \quad (10a)$$

$$g_{mn}(z)=a_{mn}\eta_{mn}\sin\left[\lambda_{mn}\left(1-\frac{z}{H}\right)\right]+c_{mn}\zeta_{mn}\sinh\left[\xi_{mn}\left(1-\frac{z}{H}\right)\right] \quad \text{(for pervious base)} \quad (10b)$$

$$\eta_i=1+\frac{1}{\varphi_i}\lambda_{mi}^2,\zeta_i=1-\frac{1}{\varphi_i}\xi_{mi}^2,\varphi_i=(n^2-1)\frac{2}{F_i}\frac{k_{hi}}{k_w}\frac{H^2}{r_e^2},\Lambda_i=\frac{k_{vi}}{m_{vi}\gamma_w},\ \Xi_{mi}=\beta_m-\frac{k_{hi}}{m_{vi}\gamma_w}\frac{2}{r_e^2F_i}\left[1+\frac{k_v}{k_w}(n^2-1)\right]$$

$$\Theta_{mi}=-(n^2-1)\frac{2}{r_e^2F_i}\frac{k_{hi}}{k_w}\beta_m,\quad \lambda_{mi}=H\sqrt{\frac{\Xi_{mi}+\sqrt{\Xi_{mi}^2-4\Lambda_i\Theta_{mi}}}{2\Lambda_i}},\xi_{mi}=H\sqrt{\frac{-\Xi_{mi}+\sqrt{\Xi_{mi}^2-4\Lambda_i\Theta_{mi}}}{2\Lambda_i}},$$

$$F_i=\left(\ln\frac{n}{s}+\frac{k_{hi}}{k_{si}}\ln s-\frac{3}{4}\right)\frac{n^2}{n^2-1}+\frac{s^2}{n^2-1}\left(1-\frac{k_{hi}}{k_{si}}\right)\left(1-\frac{s^2}{4n^2}\right)+\frac{k_{hi}}{k_{si}}\frac{1}{n^2-1}\left(1-\frac{1}{4n^2}\right),n=\frac{r_e}{r_w},s=\frac{r_s}{r_w}$$

In oder to obtain the coefficients of above equations, the three-layered system with impervious base is as an expample. Substituting Eq.(5) ~ (7) into continuous conditions, and changing to matric form:

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$$\mathbf{S}\mathbf{X}^{\mathrm{T}} = 0 \qquad (11)$$

$$\mathbf{S} = \begin{bmatrix} s_{11} & s_{12} & s_{13} & s_{14} & s_{15} & s_{16} & 0 & 0 \\ s_{21} & s_{22} & s_{23} & s_{24} & s_{25} & s_{26} & 0 & 0 \\ s_{31} & s_{32} & s_{33} & s_{34} & s_{35} & s_{36} & 0 & 0 \\ s_{41} & s_{42} & s_{43} & s_{44} & s_{45} & s_{46} & 0 & 0 \\ 0 & 0 & s_{53} & s_{54} & s_{55} & s_{56} & s_{57} & s_{58} \\ 0 & 0 & s_{63} & s_{64} & s_{65} & s_{66} & s_{67} & s_{68} \\ 0 & 0 & s_{73} & s_{74} & s_{75} & s_{76} & s_{77} & s_{78} \\ 0 & 0 & s_{83} & s_{84} & s_{85} & s_{86} & s_{87} & s_{88} \end{bmatrix} \qquad (12)$$

$$\mathbf{X} = \begin{bmatrix} 1 & c_{m1} & a_{m2} & b_{m2} & c_{m2} & d_{m2} & b_{m3} & d_{m3} \end{bmatrix} \qquad (13)$$

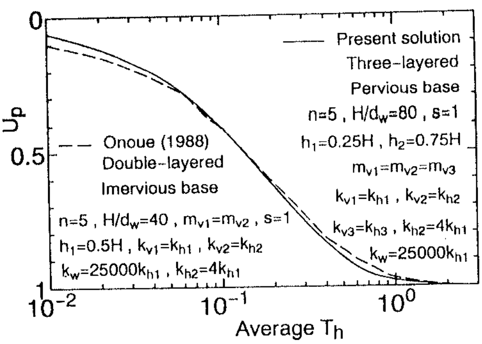

Fig. 2 Comparison of overall average consolidation degree for whole soil layer

$s_{11} = \sin(\lambda_{m1}\rho_1)$ $\quad s_{12} = \sinh(\xi_{m1}\rho_1)$

$s_{13} = -\sin(\lambda_{m2}\rho_1)$ $\quad s_{14} = -\cos(\lambda_{m2}\rho_1)$

$s_{15} = -\sinh(\lambda_{m2}\rho_1)$ $\quad s_{16} = -\cosh(\xi_{m2}\rho_1)$ $\quad s_{21} = \eta_{m1}s_{11}$ $\quad s_{22} = \eta_{m1}s_{12}$

$s_{23} = \eta_{m2}s_{13}$, $\quad s_{24} = \eta_{m2}s_{14}$, $\quad s_{25} = \zeta_{m2}s_{15}$ $\quad s_{26} = \zeta_{m2}s_{16}$

$s_{31} = \lambda_{m1}\cos(\lambda_{m1}\rho_1)$ $\quad s_{32} = \xi_{m1}\cosh(\xi_{m1}\rho_1)$ $\quad s_{33} = -\lambda_{m2}\sin(\lambda_{m2}\rho_1)$ $\quad s_{34} = -\lambda_{m2}\cos(\lambda_{m2}\rho_1)$,

$s_{35} = -\xi_{m2}\sinh(\xi_{m2}\rho_1)$ $\quad s_{36} = -\xi_{m2}\cosh(\xi_{m2}\rho_1)$ $\quad s_{41} = k_{v1}\eta_{m1}s_{31}$ $\quad s_{42} = k_{v1}\eta_{m1}s_{32}$,

$s_{43} = k_{v2}\eta_{m2}s_{33}$ $\quad s_{44} = k_{v2}\eta_{m2}s_{34}$ $\quad s_{45} = k_{v2}\zeta_{m2}s_{35}$ $\quad s_{46} = k_{v2}\zeta_{m2}s_{36}$

$s_{53} = \sin(\lambda_{m2}\rho_2)$ $\quad s_{54} = \cos(\lambda_{m2}\rho_2)$ $\quad s_{55} = \sinh(\xi_{m2}\rho_2)$ $\quad s_{56} = \cosh(\xi_{m2}\rho_2)$

$s_{57} = -\cos\left[\lambda_{m3}(1-\rho_2)\right]$ $\quad s_{58} = -\cosh\left[\xi_{m3}(1-\rho_2)\right]$ $\quad s_{63} = \eta_{m2}s_{53}$ $\quad s_{64} = \eta_{m2}s_{54}$,

$s_{65} = \zeta_{m2}s_{55}$, $\quad s_{66} = \zeta_{m2}s_{56}$, $\quad s_{67} = \eta_{m3}s_{57}$, $\quad s_{68} = \zeta_{m3}s_{58}$

$s_{73} = \lambda_{m2}\sin(\lambda_{m2}\rho_2)$ $\quad s_{74} = \lambda_{m2}\cos(\lambda_{m2}\rho_2)$ $\quad s_{75} = \xi_{m2}\sinh(\xi_{m2}\rho_2)$ $\quad s_{76} = \xi_{m2}\cosh(\xi_{m2}\rho_2)$

$s_{77} = -\lambda_{m3}\sin\left[\lambda_{m3}(1-\rho_2)\right]$ $\quad s_{78} = \xi_{m3}\sinh\left[\xi_{m3}(1-\rho_2)\right]$ $\quad s_{83} = k_{v2}\eta_{m2}s_{73}$ $\quad s_{84} = k_{v2}\eta_{m2}s_{74}$,

$s_{85} = k_{v2}\zeta_{m2}s_{75}$ $\quad s_{76} = k_{v2}\zeta_{m2}s_{76}$ $\quad s_{87} = k_{v3}\eta_{m3}s_{77}$ $\quad s_{88} = k_{v3}\zeta_{m3}s_{78}$ $\quad \rho_i = h_i/H$

In order to get unequal zero solutions of $\mathbf{X}$, ordering $\mathbf{S} = \mathbf{0}$, o, β_m is obtained. nd then, substituting β_m into Eq.(11), and $\mathbf{X}$ is obtained.

By the virtue of orthogonality of system, we get:

$$A_m = \sum_{i=1}^{n} m_{vi}\int_{h_{i-1}}^{h_i} u_0(z)g_{mi}(z)dz \Big/ \sum_{i=1}^{n} m_{vi}\int_{h_{i-1}}^{h_i} g_{mi}^2(z)dz \qquad (14)$$

The average consolidation degree at any depth is:

$$U_i(z) = 1 - \frac{\bar{u}_i}{u_0} = 1 - \frac{1}{u_0}\sum_{m=0}^{\infty} A_m g_{mi}(z)e^{-\beta_m t} \qquad (15)$$

The overall average consolidation degree for whole soil layer is defined as pore pressure and settlement, respectively:

$$\overline{U}_p = 1 - \frac{1}{u_0 H}\sum_{i=1}^{n}\sum_{m=0}^{\infty} A_m \int_{h_{i-1}}^{h_i} g_{mi}(z)dz \quad (16) \qquad \overline{U}_s = 1 - \sum_{i=1}^{n} m_{vi}\int_{h_{i-1}}^{h_i} g_{mi}(z)dz \Big/ u_0\sum_{i=1}^{n} m_{vi}(h_i - h_{i-1}) \quad (17)$$

COMPARING WITH NUMERICAL SOLUTIONS

In Fig.2, double-layered system with impervious base can be regarded as three-layered system with pervious base and symmetry with respect to the middle of whole soil layer.

CONCLUSIONS

1). The analytical solution for multi-layered ground with vertical drains is obtained.

2). The difference between the results by the present analytical solution and by numerical solutions is small.

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