III - A 395 ON THE VARIATION OF POISSON'S RATIO IN SHALLOW SOIL LAYERS

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INTRODUCTION

In geotechnical engineering Poisson's ratio is of importance in modeling the elastic behavior of soils. It is the ratio of transverse to longitudinal strain under an applied stress and can be determined from the ratio of compressional wave velocity to shear wave velocity. Due to the existence of ground water, soils are often present in the form of porous solids saturated by fluid in nature. They can be more realistically described as a two-phase material, the behavior of which is quite different from that of single phase solids. For saturated soils, there exist two types of Poisson's ratios, namely, undrained and drained Poisson's ratios, which are related to that the soils are deformed under undrained and drained conditions, respectively. However, in certain situations soils are not fully saturated. For example partially saturated conditions may be caused by fluctuating water tables and recharge of groundwater. Some in-situ tests have shown that the shallow soil layers below water table are often characterized by low velocity and high attenuation of P-wave, which is believed to be caused by a small amount of air contained in the soils(Nakamura et al 1991). Even for such nearly saturated soils, the dynamic response differs considerably from that of saturated soils due principally to the compressibility of the pore fluid(Yang and Sato 1998).

In this paper the expressions for evaluating two types of Poisson's ratios for soils are presented based on the theory of wave propagation in saturated porous media. The two-phase behavior of soils is represented by the complete Biot's formulation, which takes account not only the compressibilities of solid and fluid constituents, but the mass and viscous couplings also. The effect of a small amount of air dissolved in the shallow layers on Poisson's ratio is examined. On such a basis, a conceptual description of the possible structures of Poisson's ratio and wave velocity in shallow soil layers is proposed.

POISSON'S RATIO FOR NEARLY SATURATED SOILS

In shallow saturated soils, there may exist a small amount of air which is dissolved in pore water. For such a case of high saturation, the concept of "an homogeneous fluid" can be introduced. The bulk modulus of the homogeneous fluid D_f depends on the saturation S_r as

$$D_f = \frac{1}{\dfrac{1}{K_f} + \dfrac{1-S_r}{p}} \qquad (1)$$

where K_f is the bulk modulus of water and p is the absolute fluid pressure.

According to Biot's theory, the equations of motion for two-phase media can be given as(Biot 1962)

$$G\nabla^2\vec{u} + (\lambda_c + G)\nabla e - \alpha M\nabla\zeta = \rho\ddot{\vec{u}} + \rho_f\ddot{\vec{w}} \qquad (2)$$

$$\alpha M\nabla e - M\nabla\zeta = \rho_f\ddot{\vec{u}} + m\ddot{\vec{w}} + b\dot{\vec{w}} \qquad (3)$$

where $e = \mathrm{div}\vec{u}$, $\zeta = -\mathrm{div}\vec{w}$, $\vec{u}$ and $\vec{w}$ are the displacement vectors of the solid skeleton and the fluid with respect to the solid phase; m and b describe the mass and viscous couplings; ρ is the total density, $\rho = (1-n)\rho_s + n\rho_f$, ρ_s is the mass density of the grains; λ and G are Lame constants of soil skeleton, $\lambda_c = \lambda + \alpha^2 M$, α, M are parameters accounting for the compressibilities of the grains and fluid

$$\alpha = 1 - \frac{K_b}{K_s},\ M = \frac{K_s^2}{K_d - K_b},\ K_d = K_s[1 + n(\frac{K_s}{D_f} - 1)] \qquad (4)$$

in which K_s and K_b are the bulk moduli of solid grains and skeleton, respectively.

Solving Eqs.(2,3) yields the following complex wave velocities for the two kinds of compressional waves and one shear wave(Yang and Sato 1998):

$$V_{1,2} = \sqrt{\frac{-B \pm \sqrt{B^2 - 4AC}}{2C}} \qquad V_s = \sqrt{\frac{G(ib/\omega - m)}{\rho ib/\omega - C}} \qquad (5)$$

where $A = (\lambda + 2G)M$, $B = (2\alpha\rho_f - \rho)M - (\lambda_c + 2G)m$, $C = \rho m - \rho_f^2$, $i = \sqrt{-1}$ and ω is the circular frequency.

In low frequency range, where the frequencies generally used in field tests fall, the velocity of the slow compressional wave approaches zero and the velocities of the fast compressional wave and shear wave approximately take the following forms

$$V_p = \sqrt{\frac{\lambda + 2G + \alpha^2 M}{\rho}} \qquad V_s = \sqrt{\frac{G}{\rho}} \qquad (6)$$

The drained Poisson's ratio ν_d relates to Lame constants as

$$\nu_d = \frac{\lambda}{2(\lambda + G)} \qquad (7)$$

Thus, introducing (6) into (7) yields the drained Poisson's ratio in terms of velocity ratio V_p/V_s as

$$\nu_d = \frac{1}{2}\frac{\alpha^2 M/G - (V_p/V_s)^2 + 2}{\alpha^2 M/G - (V_p/V_s)^2 + 1} \qquad (8)$$

On the other hand, in linear elasticity the relationship of velocity ratio and undrained Poisson's ratio is

$$\nu_u = \frac{1}{2}\frac{(V_p/V_s)^2 - 2}{(V_p/V_s)^2 - 1} \qquad (9)$$

With (8,9) the undrained Poisson's ratio is obtained as

$$\nu_u = \frac{1}{2}\frac{\alpha^2 M/G + 2\nu_d/(1-2\nu_d)}{\alpha^2 M/G + 1/(1-2\nu_d)} \qquad (10)$$

Following the linear poroelasticity and Skempton's relation(1954), the undrained Poisson's ratio can be also derived as

$$\nu_u = \frac{3\nu_d + B_s(1-2\nu_d)(1-K_b/K_s)}{3 - B_s(1-2\nu_d)(1-K_b/K_s)} \qquad (11)$$

where B_s is pore pressure coefficient of Skempton. It can be easily shown that (10) coincides with (11) by introducing an alternative interpretation of α, M (Senjuntichai and Rajapakse 1994). Obviously it can be shown that the practical range of ν_u is $\nu_d \le \nu_u \le 1/2$, the upper limit is reached for incompressible constituents($\alpha = 1, 1/M = 0$ or $B_s = 1, K_b/K_s = 0$)and the lower bound is achieved when pore fluid is highly compressible($D_f << G$ or $B_s = 0$).

In the case of full saturation, by ignoring the compressibility of soil grains and considering that $\lambda + 2G << K_f/n$, (10) is further simplified as

$$\nu_u \approx \frac{1}{2}(1 - \frac{nG}{K_f}) \qquad (12)$$

This approximate expression can be also obtained following the elasticity and Skempton relation as shown in Ishihara(1996).

EFFECT OF PARTIAL AIR SATURATION

The relationship of drained Poisson's ratio and velocity ratio in (8) is plotted in Fig.1 for various modulus ratios($MR = D_f/G$). The porosity is assumed to be 0.4. As can be seen, for a given Poisson's ratio, the velocity ratio increases with the modulus ratio increasing, on the other hand, for a given modulus ratio, the variation of velocity ratio is very small in the range of $\nu_d \le 0.4$, while it rises rapidly with the increase of ν_d in the range of ν_d >0.4.

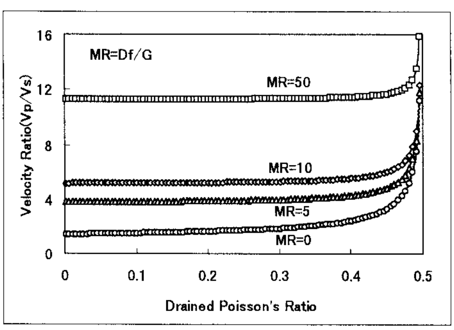

Fig.1 Velocity ratio as a function of drained Poisson's ratio

The effect of partial air saturation on the undrained Poisson's ratio is shown in Fig.2 for several soil types, the drained Poisson's ratio is taken to be 0.33 and porosity is 0.4, p in (1) is 100kPa . Clearly it is seen that the upper limit of undrained Poisson's ratio is 0.5, which may be reached for full saturation for soft soils, while the lower bound is the drained Poisson's ratio, which can be achieved for relatively high air saturation. Between the two limits, the variation of undrained Poisson's ratio is little for a very soft soil, while the transition is large for a soil with a higher stiffness.

Based on the above analysis, we see that even a small amount of air contained in soils can produce an obvious influence in the undrained Poisson's ratio. It is also known that the velocity of the first compressional wave decreases dramatically with even slight decreases below full saturation, while shear wave velocity is nearly not affected by air content(Yang and Sato 1998). Therefore, we can plot the possible structures of wave velocity and Poisson's ratio in a conceptual sense as shown in Fig.3 by considering the variation of water saturation in the vertical direction in the shallow soils. The surficial soils over the water table may be regarded as a partial saturated or dry state, the pore air is in the form of open channel. The degree of saturation of soils below the water table increases from the partial saturation to high saturation and finally to the full saturation at some depth. The velocity of fast compressional wave increases dramatically from partial saturation to full saturation, while the shear wave velocity may gradually increase with depth due principally to the increase of stiffness. As for the undrained Poisson's ratio, it could increase with depth until the soils reach the state of full saturation, after that it tends to decrease with the depth due to the increase of the stiffness of soils.

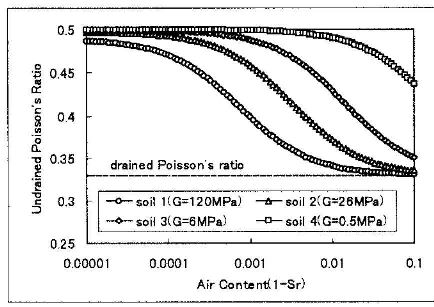

Fig.2 Effect of air content on undrained Poisson's ratio

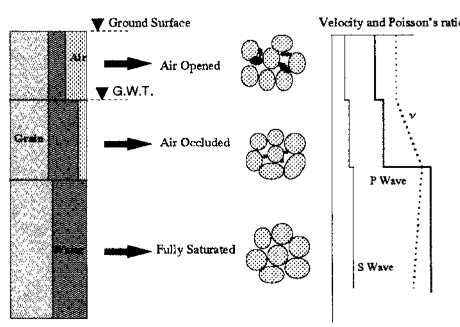

Fig.3 Possible structures of velocity and Poisson's ratio

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