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The Energy Approach in Fracture Analysis of Concrete

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1. Proposed Model

1.1 Energy criterion on crack initiation and extension

Convenient as it is though, the strength criterion may not suffice as an appropriate criterion for crack analysis of concrete, because it lacks the energy basis which states that crack extension occurs when the energy available for crack growth is sufficient to overcome the resistance of the material. However, in implementing the energy principle, there is the difficulty in determining the available energy to drive the cracks, which is not straightforward because the energy dissipation due to plasticity coexists. Therefore, in this study, instead of directly estimating the energy release rate from potential energy, the overall rate of possible energy loss with crack area $<g'_f>$ is predicted from the strain softening curve of the FPZ and viewed as the effective driving force for fracture, see Shi *et al* (1998).

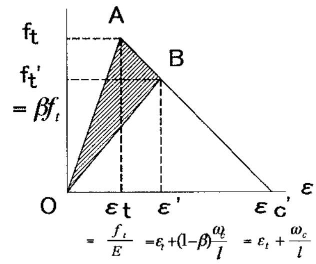

Fig 1: Energy dissipation due to cracking

Fig 1 shows a stable crack occurred when $<g'_f>=g'_{fc}$, with g'_{fc} being the material's resistance to fracture. The stress-displacement relation for the FPZ is expressed in the stress-strain form as the crack is smeared inside the effective domain of the stress point along its equivalent length l, which is the length of the domain perpendicular to the crack. For simplicity, a linear relation between the cohesive stress σ and the apparent strain ε' is assumed. Suppose that the extension of the crack with an opening displacement ω stops as the cohesive stress σ drops to f_t', with $f_t'=\beta f_t$ where f_t is the material's original tensile strength and $0<\beta<1$. Then the area ΔOAB represents the energy dissipated during the current fracturing process G'_f.

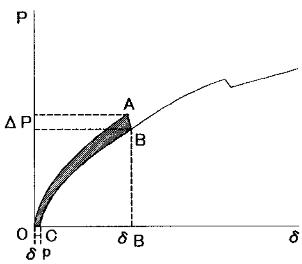

Fig 2: Loss of strain energy due to cracking

Built on the theory of homogenization, the overall rate of the energy loss with crack area over V_c, which is the potential region of cracks, is then obtained by the volume average.

$$< g'_f >= \frac{1}{V_c}\int_{V_c} g'_f dV = \sum_{\alpha=1}^{N} g'^{\alpha}_f \phi^{\alpha} \quad \text{with} \quad g'_f = \frac{G'_f}{\Delta\omega} = \frac{\Delta OAB}{(1-\beta)\omega_c}, \quad \sum_{\alpha=1}^{N}\phi^{\alpha} = 1 \qquad (1)$$

where g'^{α}_f is the rate of energy loss for crack α, and $\phi^{\alpha}(=V^{\alpha}/V_c)$ represents its volume fraction with V^{α} being the domain in which g'^{α}_f is assumed constant, and N is the total number of potential cracks. Note that a crack will develop when the tensile stress computed in the local Cartesian coordinate system reaches the corresponding local tensile strength f_t' of the material, and is termed as a potential crack before an actual crack analysis is carried out. In the numerical approach, a virtual crack analysis is performed at each load increment so that the overall rate of the energy loss $<g'_f>$ can be predicted and compared with the structural resistance to fracture g'_{fc}. At the moment of fracture, $< g'_f >= g'_{fc}$, and the actual crack analysis is then executed by the smeared crack approach with the E-ω relation, see Shi *et al* (1998).

1.2 Balance between fracture energy and strain energy

Abrupt load drop simultaneous to the occurrence of stable cracks is a well-observed phenomenon of various kinds of

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fracture tests on concrete specimens. This is, of course, due to the dissipation of fracture energy by each individual crack which in turn precipitates the loss of the structure's strain energy. For simplicity, consider only a single concentrated load acting on a concrete structure. Fig. 2 shows the load-deflection curve. The shaded area *OABC* represents the total strain energy loss when stable cracks occur or extend at the load level P. Of which a small fraction is caused by the material's
plasticity. Subtracting this portion from *OABC*, the strain energy loss caused solely by cracking ΔU_f is evaluated. The energy dissipation W_f during the current process of fracturing is given by integrating G_f' over the entire fractured area V_c:

The new unstable equilibrium position B after cracking (see Fig. 2) is obtained numerically for a number of small negative load increments until $\Delta U_f = W_f$ is satisfied. Sum of these small negative load increments yields the load decrease ΔP. Figure 3 illustrates the main steps of fracture analysis used in the present approach.

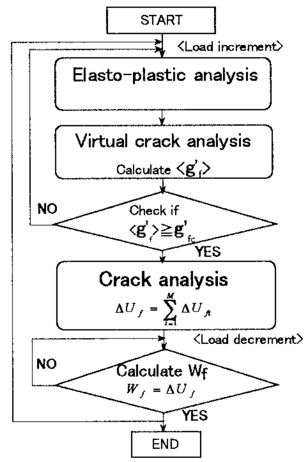

Fig. 3. Flow for nonlinear analysis based on energy approach

2. Comparison with Experimental Results

Experimental results for plain concrete beams involved data in reference to the results of round robin flexural tests of concrete beams, see Report of Committee of Application of Fracture Mechanics to Concrete Structures (1993). Fig. 4 shows the analyzed cases and the load-displacement relations. The loads at crack initiation and failure are reasonably agreed with the experimental results. Though the structural response of plain concrete beams is rather unstable after the critical load, the proposed approach using the load-control method can still trace this unstable behavior up to a certain point even when the snap-back instabilities are involved

3. Conclusions

A numerical approach for computational crack analysis of concrete is proposed, which is based on the overall rate of the energy loss per unit crack area. The validity of the criterion has been proved by early numerical studies, though further verification is still necessary. Next, the energy balance during a fracturing process is discussed and an equation is built for the dissipated fracture energy and the decrease in strain energy after cracking. Based on this relation, numerical approaches using load-control method become a straightforward process even when the snap-back instabilities are involved. The resulting model has been proved successful, in the sense that the finite element analyses do not diverge prematurely, and is capable of predicting plain concrete behavior as have been shown by a number of analyses on plain concrete beams.

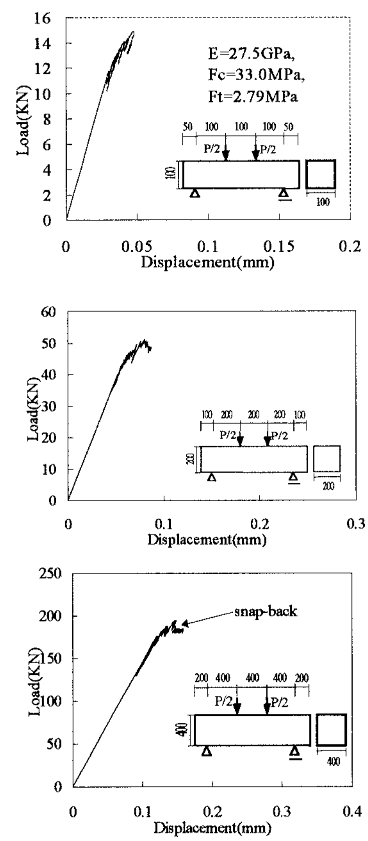

Fig. 4. Load-displacement relation

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