

# I - A229 APPLICATION OF RBSM TO A PRESTRESSED PRECAST STRUCTURE

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**1. INTRODUCTION:** Precast concrete members tightened by external and/or internal prestressing have been often used. A precast prestressed members can not be regarded as a perfectly continuous structures and shows nonlinear behavior due to not only the nonlinearity of the materials but also to the opening of the member-joint and it is very important to predict the limit states of the imperfectly continuous structures as exactly as possible.

the purpose of this study is to analyze a precast prestressed structure using the Rigid Bodies Spring Model<sup>1</sup> (RBSM) where the prestressing cables and the deviators were modeled and introduced as beam elements. The analytical results was compared with an experimental results<sup>2</sup> to confirm the idea of this model.

## 2. ANALYTICAL METHOD:

**2.1 Two Dimensional RBSM:** The RBSM proposed by Kawai, in which the structure is idealized as an assemblage of rigid body elements connected by two kinds of distributed spring as shown in Fig. 1. So the transmission of internal force and the deformation of a body a represented by the spring system. An arbitrary point P of the edge AC before deformation will be displaced to P' of AC and P'' of A'C' respectively after deformation. the relative displacement between P' and P'' denoted in the local coordinate system by  $\{\delta\} = \{\delta_d, \delta_n\}^T$  can be given in by the centroidal displacement of each element denoted in the global coordinate system as  $\{u\} = [u_1, v_1, \theta_1, u_2, v_2, \theta_2]^T$

$$\{\delta\} = [B]\{u\}, \quad [B] = [M][R][Q] \quad (1)$$

where [Q] is a matrix to derive the displacements of points P' and P'' from the centroidal displacement, [R] is a coordinate transformation matrix and [M] is a matrix to derive the relative displacement from P' and P''.

The spring constants  $k_d$  and  $k_n$  on the contact surface between element ① and ② can be determined by using the finite difference equation for strain components as follows:

$$\{\epsilon\} = \begin{Bmatrix} \epsilon_d \\ \gamma_n \end{Bmatrix} = \frac{1}{h_1 + h_2} \begin{Bmatrix} \delta_d \\ \delta_n \end{Bmatrix} = \frac{1}{h} \{\delta\} \quad (2a)$$

$$\{\sigma\} = \begin{Bmatrix} \sigma_d \\ \tau_n \end{Bmatrix} = \begin{Bmatrix} \frac{(1-\nu)E}{(1-2\nu)(1+\nu)} \epsilon_d \\ \frac{E}{(1-\nu)} \gamma_n \end{Bmatrix} = \begin{bmatrix} k_d & 0 \\ 0 & k_n \end{bmatrix} \begin{Bmatrix} \delta_d \\ \delta_n \end{Bmatrix} = [D]\{\delta\} \quad (2b)$$

where E is the Young's modulus,  $\nu$  is the Poisson ratio,  $h = h_1 + h_2$ , and  $h_1$  and  $h_2$  are the distances between the edge AC and the centroid of respective elements before deformation.

**2.2 Modelization of Tendons and Deviators Using Beam Element:** The beam element model was used to introduce the prestressing tendons and deviators into the RBSM. this model considers the deformation of 2 rigid bars connected by a system spring as shown in Fig. 2. Using the same process described above, the spring constants  $k_d$  and  $k_n$  and the strain energy V can be given as follows:

$$k_d = \frac{E}{l}, \quad k_n = \frac{E}{2(1+\nu)L} \quad \text{where } L = L_1 + L_2/2$$

The models proposed for the external tendons+deviators, and for the internal tendons are shown respectively in Fig.3 and Fig. 4., Fig. 5 shows the materials characteristics used in this analysis.

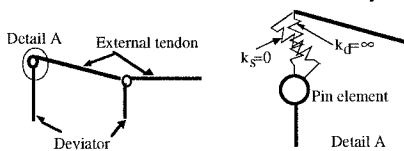


Fig. 3 Assumed Model for External Tendon

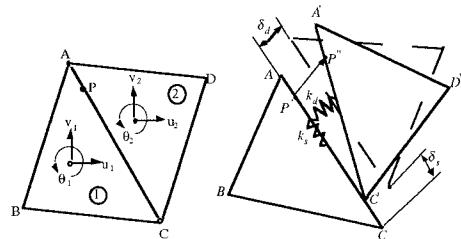


Fig. 1 Rigid Triangular Elements

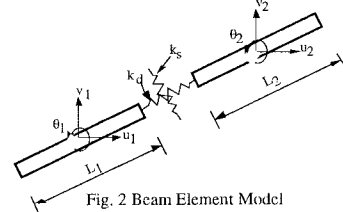


Fig. 2 Beam Element Model

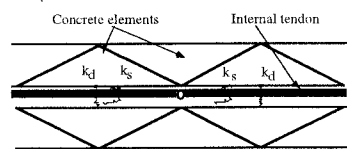


Fig. 4 Assumed Model for Internal Tendon

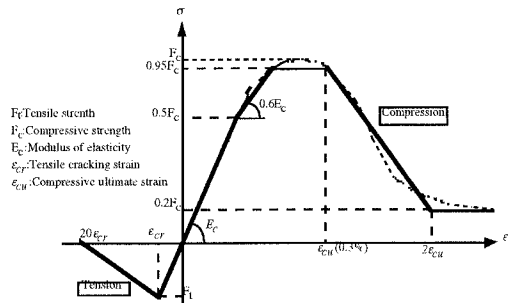


Fig. 5 Materials Characteristics

**Key Words:** External tendon, Beam Element

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### 3. RESULTS AND DISCUSSION

The analysis was made for 3 different cases, internal tendons only (case 1), internal and external tendons (case 2) and using external tendons after the first crack (case 3). All the results are compared with the experimental results. For the three cases, Fig. 6 shows the load-displacement relation, Fig. 7 shows the opening of the joints close to the center of the span and Fig. 8 and Fig. 9 show the strain behavior of the bottom flange, it is clearly that the analytical results are in good agreement with the experimental data. Also the effect of the reinforcement of the structure with external tendons, in case 3 it can be observed.

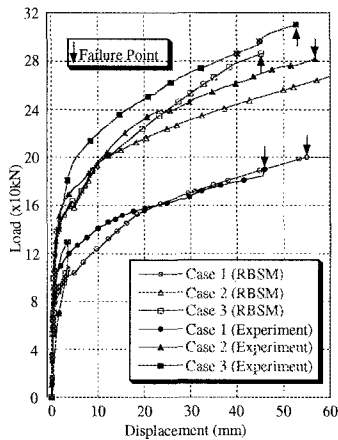


Fig. 6 Load-Displacement Behavior

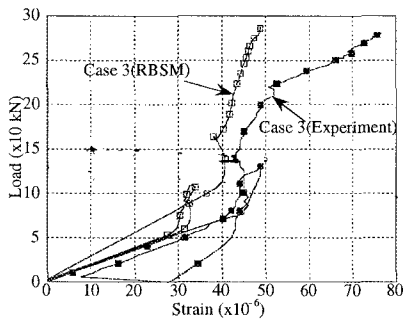


Fig. 9 Load-Strain Behavior (Tension Side)

Also it can be observed in Fig. 10 (a) and (b) that the analysis and experiments gave almost the same crack pattern.

### 4. CONCLUSIONS

The analytical investigations, using the RBSM, in the flexural behavior of a precast concrete beam prestressed with internal and/or external tendons were carried out. The analytical results were in good agreement with the experimental results and confirm the idea of introducing the prestressing tendons and the deviators as beam elements. The RBSM is an effective method to analyze the behavior of a precast structure prestressed with external tendons.

### References

- 1) Kawai, T., New Element Models in Discrete Structural Analysis, Journal of the Society of Naval Architects of Japan, No.141, pp. 187-193, 1977.
- 2) Zardoum, H. et al., Flexural Behavior of a Precast Member Reinforced with External Tendons, Proceeding of 22nd International Conference on Our World in Concrete & Structures, 25-27 August 1997, Singapore, pp. 393-400.

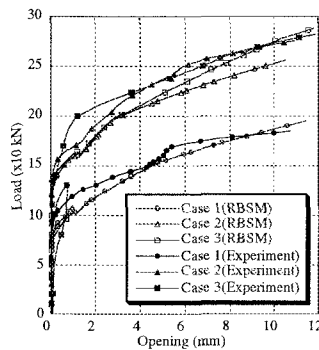


Fig. 7 Load-Opening Relation

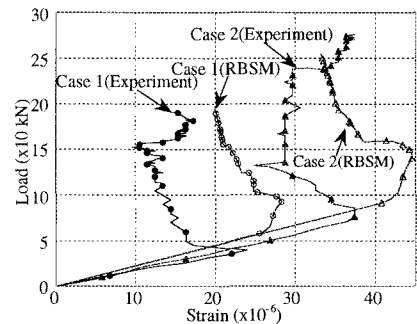
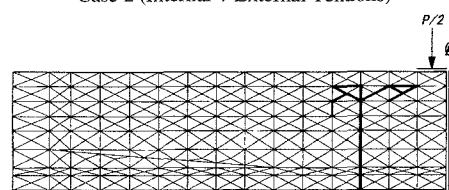
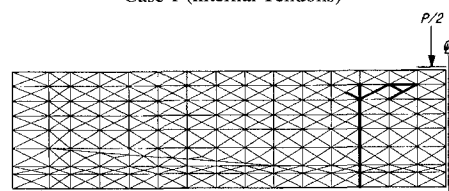
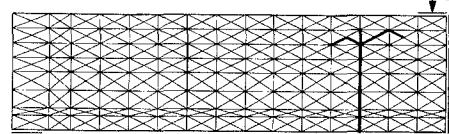
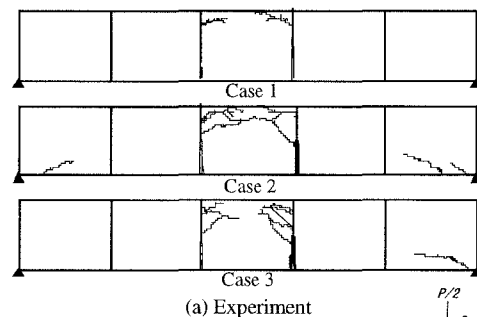


Fig. 8 Load-Strain Behavior (Tension Side)



(a) Analysis  
Fig. 10 Crack Layout