

# I - A227 APPLIED ELEMENT SIMULATION OF LARGE DEFORMATION OF STRUCTURES

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**1. Introduction** A new method for large deformation analysis of structures is proposed. The structure is modeled as an assembly of distinct elements made by dividing the structural members virtually. These elements are connected by distributed springs in both normal and tangential directions. This method depends mainly on calculating residual forces acting on each element due to geometrical changes of structure during loading. The accuracy of the model was verified in the range before rigid body motion starts<sup>1)</sup>. This paper introduces a new technique to deal with buckling and post buckling behavior of structures. Although the technique proposed is simple, results with high accuracy can be obtained in calculating the buckling loads and following post-buckling deformations.

**2. Element formulation** We assume that the two elements shown in Fig. 1 are connected by distributed normal and shear springs at contact points. Each pair of springs fully represent deformation and failure of a certain area. The formulation and results of the element before rigid body motion stage were introduced in Ref. (1) and it was proved that the method could determine deformations and detect the initiation and propagation of cracks. To develop the methodology for static large deformation analysis, the following steps are proposed.

The general equation of motion under static loading is:

$$[K][\Delta U] = \Delta f + R_m + R_G \quad (1)$$

Where  $[K]$  is nonlinear stiffness matrix,  $\Delta f$  is incremental applied load vector and  $[\Delta U]$  is incremental displacement vector. The term,  $R_m$ , is residual force vector due to cracking or incompatibility between strains and stresses at the spring location, while  $R_G$  is residual forces due to geometrical changes of the structure during loading. With this technique, we don't have to determine the geometrical stiffness matrix resulting in making the method general and applicable for any case of loading or the type of structures. The method is applied by the following steps:

1. Assume that  $R_m$  and  $R_G$  are zeros and solve the equation to get incremental displacement.
2. Modify the geometry of the structure according to the calculated incremental displacements.
3. Modify the direction of spring force vectors according to the new element configuration. Incompatibility between applied forces and internal stresses occurs due to geometrical changes.
4. Check the situation of cracking and calculate the material residuals load vector  $R_m$ .
5. Calculate the element force vector from surrounding springs of each element  $F_m$ .

6. Calculate the geometrical residuals around each element from the equation below.

$$R_G = f - F_m \quad (2)$$

**Equation (2)** above means that the geometrical residuals account for the incompatibility between external applied forces and internal forces due to modification of geometry of the structure. Small deformations are assumed during each increment.

7. Calculate the stiffness matrix for the structure in the new configuration considering stiffness changes at each spring location due to cracking or yield of reinforcement.
8. Apply again a new load or displacement increment and repeat the whole procedure.

Residuals calculated from the previous increment can be incorporated in solution of **Eq. (1)** to reduce the time of calculation.

3. **Numerical results** To check the accuracy of the newly proposed method, large deformation analyses of two case studies are introduced.

The first case is buckling and post buckling behavior of a fixed base elastic cantilever under axial load. The load direction is assumed constant during analysis. The height of the cantilever is 12.0 m and the depth is 1.0 m. The analysis was performed using 300 elements. The load was applied at the top of the column with the constant-rate vertical displacement. To break symmetry of the system, the stiffness of one of edge elements was increased by just 1% relative to the other elements. **Figure (2)** illustrates the deformed shape of the cantilever during and after buckling. **Figure (3)** shows the horizontal and vertical displacements at the loading point in two cases with and without consideration of the geometrical residuals. The calculated buckling load without the consideration of geometrical residuals, only with modification of geometry, was about 47 tf which is quite larger than the theoretical one (7.8 tf). While in case with geometrical residuals, both vertical and horizontal displacements increase drastically when the load reaches the theoretical buckling load. When the vertical displacement is about 9 m, horizontal displacement begins to decrease. Referring to **Figs. (2) and (3)**, the cantilever shape changes to an arch which makes the stiffness of the specimen increases after buckling. **Figure (4)** shows the load-stress relation of the point "A" under the applied load. Before buckling, stress is mainly compression and increases in a linear way. When reaching the buckling load, compression stresses are released till reaching zero when the direction of load becomes parallel to the cantilever end edge. Finally, tension stresses develop and

increase till the end of analysis.

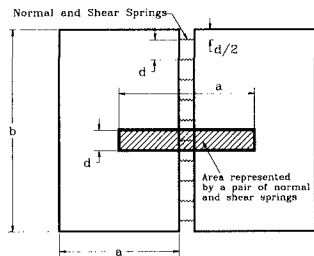
Changes in internal stresses of an intermediate section during analysis are shown in Fig. (5). Before buckling, stresses are mainly compression. After reaching the buckling load, inspite the applied load is constant ( $P=7.8\text{tf}$ ), buckling bending moments generates.

The second case study is simulation of buckling behavior of elastic frame. Base supports of the frame are fixed and two vertical loads are applied at corners. Figure (6) shows the frame shapes at initial and during buckling, and load-displacement relation. During simulation, side sway is permitted. The theoretical buckling load is very close to the calculated one.

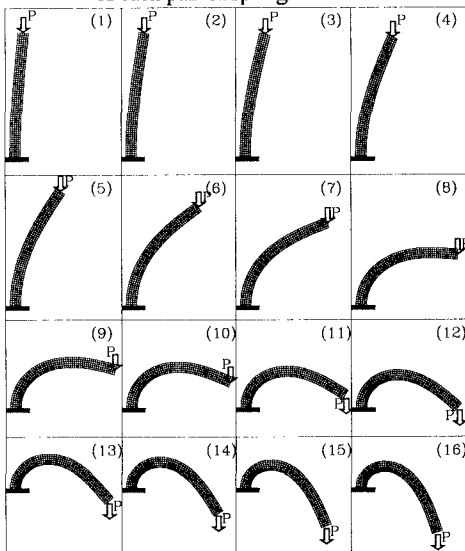
**4. Conclusions** In this study, a new technique was developed by which structure behavior can be simulated even when large geometrical changes occur. The calculated buckling loads, buckling modes and internal stresses agree well with the theoretical values. This technique can be extended easily to follow large deformation of structures till total collapse.

## References

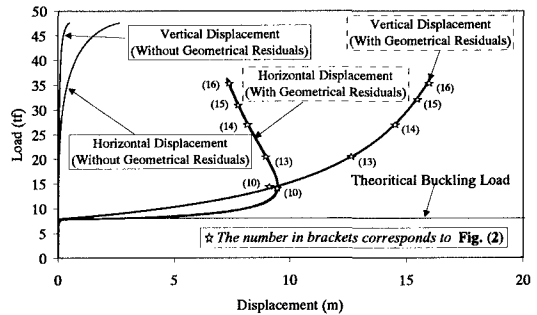
- 1) Meguro K. and Tagel-Din H.: A new efficient technique for fracture analysis of structures, Bulletin of Earthquake Resistant Structure, IIS, University of Tokyo, No. 30, pp. 103-116, 1997.



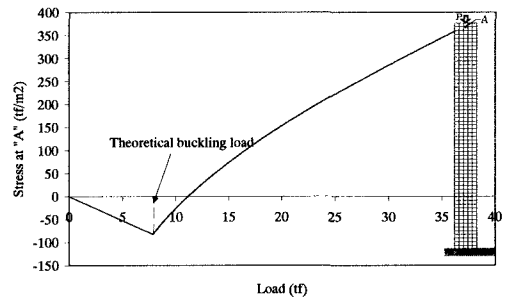
**Fig. 1** Spring distributions and area of influence of each pair of springs



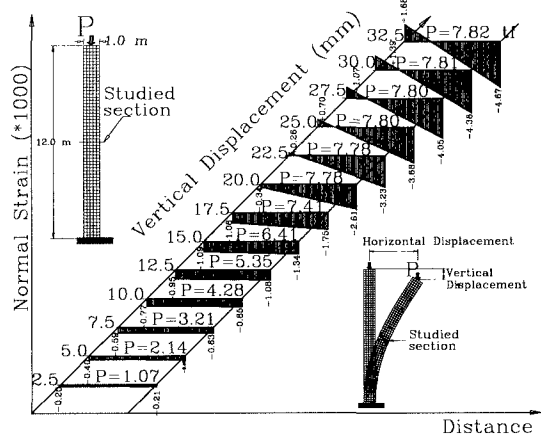
**Fig. 2** Post buckling behavior of a cantilever



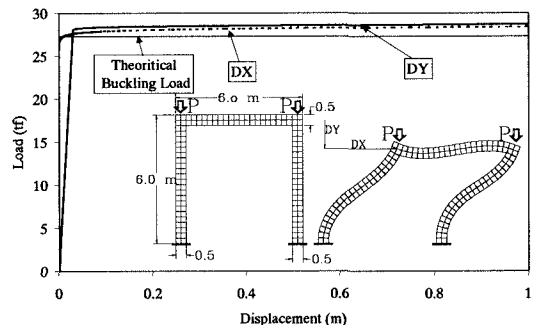
**Fig. 3** Load-displacement relation



**Fig. 4** Load-stress relation at point "A"



**Fig. 5** Variation of internal stress distribution during buckling



**Fig. 6** Load-deformation and buckling shape of a frame