

## Multicriteria Fuzzy Optimum Decision Making for Prestressed Concrete Bridge System Considering Cost and Aesthetic Feeling

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### 1. Introduction and formulation of optimum design problem of bridge system

In the practical structural design problem, the designers should take into account several objective functions, such as economics, safety, serviceability, aesthetic feeling and so on, and relative evaluations among these different characteristic objectives have some tolerances or fuzziness. For these reasons, the practical optimum structural design process concerning multiobjectives can be defined as a kind of compromising optimum decision-making process with fuzziness. In this paper, a rational, systematic and efficient fuzzy optimum design method for a three-span continuous prestressed concrete box girder bridge system shown in Fig.1 is developed by combining suboptimization concept and fuzzy decision making techniques.

In the optimum design problem of prestressed concrete bridge system, the total construction cost of bridge system  $f_c$  to be minimized and the aesthetic feeling  $f_a$  to be maximized are considered as the objective functions. For the reason that the span ratio  $Sr$  and girder height  $H$  affect significantly to these two objectives, design variables  $Sr$  and  $H$  are considered as the common design variables of bridge system. The multiobjective optimization problem of prestressed concrete bridge system, is then, formulated as

find  $X = (X_{sup}, X_{sub}, Sr, H, )$  which

$$\text{minimize } f_c(X) = W_{sup}(X_{sup}, Sr, H) + W_{sub}(X_{sub}, Sr, H) \quad (1)$$

$$\text{maximize } f_a(Sr, H) \quad (2)$$

$$\text{subject to } g_j((X_{sup} \text{ or } X_{sub}), Sr, H) \leq 0 \quad (j = 1, \dots, q) \quad (3)$$

where  $X$  and  $W$  are, respectively, the vector of design variables and construction cost. Subscripts sup and sub denote, respectively, superstructure and substructure.  $g_j$  is the  $j$ th design constraints to be taken into accounts for superstructure or substructures.  $q$  is the total number of design constraints.

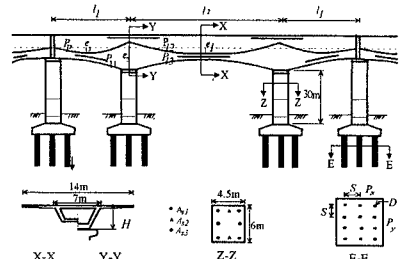


Fig. 1. Design variables of three-span continuous prestressed concrete box girder bridge system

### 2. Suboptimizations of bridge system and fuzzy membership functions of objective functions

At the first stage of optimum design method, the cost minimization problem of superstructure and substructures including piers and pile foundations of bridge system are suboptimized for the discrete combinations of common design variables  $Sr$  and  $H$ . In the optimum design of superstructure, the parabolic prestressing force  $P_p$ , linear partial prestressing forces  $P_l$ , thickness of bottom slab of box section  $t$  and tendon eccentricities of parabolic prestressing  $e$  are assumed as the design variables as shown in Fig. 1,  $X_{sup} = ([P_p, P_{l1}, P_{l2}, P_{l3}, e_1, e_2, e_3, t]^T)$ , and stress and cracking constraints in the serviceability limit state and flexural-strength and ductility constraints in the ultimate limit state specified in the ACI code are taken into account. The minimum cost design problem of superstructure is solved by an optimal design method combining the convex approximation concept and dual method.

In the minimum design of substructures, each pier is assumed to be consisted of three segments with same widths and depths from the aesthetic viewpoint. Then only the reinforcement areas in each pier segment shown in Fig. 1, are dealt with as the design variables. The minimum cost problem of RC pier segment is solved also by the dual method subject to the ultimate limit state constraints under vertical force and bending moments due to horizontal forces at earthquake. In the minimum cost design problem of RC pile foundations, number of RC piles in the direction of bridge axis,  $P_x$ , and that in the perpendicular direction,  $P_y$ , diameter of pile,  $D$ , and interval of piles,  $S$ , in each pile foundation are dealt with as the design variables as shown in Fig. 1. A RC pile is optimized subject to the constraints on bearing or tensile capacities of pile. The optimum values of  $P_x$ ,  $P_y$ ,  $D$  and  $S$  are determined by applying a systematic iterative and comparing process for discrete sets of these design variables. As a result of the above suboptimization process, the minimum total construction cost of bridge system versus  $Sr$  and  $H$  relationships are introduced.

At the next stage, a measure of membership function of minimum total construction cost is introduced by inspecting the financial tolerance for total construction cost and the range of variation of the suboptimized minimum total construction costs of the bridge system at all discrete sets of common design variables  $Sr$  and  $H$ . Membership functions of minimum total construction cost with respect to girder height  $H$  at various discrete span ratios  $Sr_k$ ,  $\mu_i(H, Sr_k)$  are obtained by using the above measure of membership function of total construction cost and the suboptimized minimum total construction costs at discrete combinations of  $Sr$  and  $H$ . The membership functions of aesthetic feeling with respect to web height  $H$  at every discrete span ratios,  $Sr_k$ ,  $\mu_i(H, Sr_k)$  can be introduced by evaluating relative aesthetic feelings of perspective views of bridge system with various discrete sets of common design variables  $Sr$  and  $H$  in the surroundings at construction site.

Prestressed Concrete Bridge, Fuzzy Optimization, Suboptimization, Total Construction Cost, Aesthetic Feeling  
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### 3. Determination of the fuzzy optimum solution

By using the membership functions of minimum total construction cost  $\mu_t(H, Sr_k)$  and aesthetic feeling  $\mu_a(H, Sr_k)$ , we can determine the fuzzy optimum solution by the minimum operator method or weighted operator method.

According to the minimum operator method, the maximum membership value  $\mu_{k,opt}$  at the  $k$ th discrete span ratio  $Sr_k$  can be determined by the following expression.

$$\mu_{k,opt}(H_{k,opt}, Sr_k) = \max\{\min[\mu_t(H, Sr_k), \mu_a(H, Sr_k)]\} \quad (4)$$

Then we can introduce the relationship between maximum membership value  $\mu_{k,opt}(H_{k,opt}, Sr_k)$  with respect to span ratio  $Sr$  and the final global optimum span ratio  $Sr_{opt}$  is determined as that has the maximum membership value  $\mu_{opt}$ . The optimum girder height  $H_{opt}$  at  $Sr_{opt}$  can be determined by estimating the corresponding relationship between membership values versus  $H$  at  $Sr_{opt}$ . The exact global optimum values of  $X_{sup}$  and  $X_{sub}$  at  $Sr_{opt}$  and  $H_{opt}$  are determined by the suboptimization process described previously.

In the weighted operator method, membership functions of minimum total construction cost and aesthetic feeling are multiplied by the normalized relative weights  $W_t$  and  $W_a$ , where  $W_t + W_a = 1.0$ , which are determined by the designer's preference and design emphases of the structure. Then, the maximum membership value  $\mu_{k,opt}$  and the optimum girder height  $H_{k,opt}$  at the  $k$ th span ratio can be determined by the following expression.

$$\mu_{k,opt}(H_{k,opt}, Sr_k) = \max\{W_t \mu_t(H, Sr_k) + W_a \mu_a(H, Sr_k)\} \quad (5)$$

The relationship between weighted maximum membership value  $\mu_{k,opt}(H_{k,opt}, Sr_k)$  and span ratio  $Sr$  is introduced by the same process in the minimum operator method and the final global optimum span ratio  $Sr_{opt}$  and girder height  $H_{opt}$  and exact values of  $X_{sup}$ ,  $X_{sub}$  at  $Sr_{opt}$  and  $H_{opt}$  are determined also by the same process in the minimum operator method.

### 4. Numerical design example and concluding remarks

As the numerical design example, the bridge system with total length 200m is illustrated. Span ratios  $Sr = 0.5, 0.61, 0.75, 0.92$  and web heights at interior support  $H = 4.5\text{m}, 5.0\text{m}, 5.5\text{m}, 6.0\text{m}, 6.5\text{m}, 7.0\text{m}, 7.5\text{m}, 8.0\text{m}, 8.5\text{m}$  are considered as the discrete values of common design variables. At the first stage of the optimization process, the superstructure and substructures are suboptimized at the various discrete set of  $Sr$  and  $H$ . As a result, the minimum total construction cost with respect to girder height  $H$  relationship for  $Sr=0.5, 0.61, 0.75, 0.92$  are introduced. Fig. 2 shows the two examples of these relationships for  $Sr=0.5$  and  $0.75$ .

At the next stage, the measure of membership function of minimum total construction cost is introduced as depicted in Fig. 3. As the measure of membership function of aesthetic feeling of bridge system, the bridge system with  $Sr=0.61$ ,  $H=6.5\text{m}$  is decided as the most beautiful bridge system giving the best harmony with surrounding situation at construction site and the measure of membership function of aesthetic feeling is decided as depicted in Fig. 4.

By using above two measures of membership functions, the membership functions of minimum total construction cost and aesthetic feeling at discrete  $Sr_k$ ,  $\mu_t(H, Sr_k)$  and  $\mu_a(H, Sr_k)$ , are introduced. Substituting these membership functions into eq. 4 of the minimum operator method, we can determine the maximum membership value  $\mu_{k,opt}(H_{k,opt}, Sr_k)$  for each discrete  $Sr$ . The final global optimum span ratio  $Sr_{opt} = 0.63$  is obtained from Fig. 5. The optimum girder height  $H_{opt}$  is determined from the relationship between membership values versus  $H$  at  $Sr_{opt}=0.63$ .

In the weighted operator method, the membership functions of minimum total construction cost and aesthetic feeling for each discrete  $Sr_k$  are multiplied by the normalized relative weights  $W_t=0.6$  and  $W_a=0.4$ . Substituting these membership functions into eq. 5 of the weighted operator method, we can determine the maximum membership value  $\mu_{k,opt}(H_{k,opt}, Sr_k)$  for each discrete  $Sr_k$ . The final global optimum span ratio  $Sr_{opt}=0.74$  is obtained from Fig. 5.  $H_{opt}=7.2\text{m}$  is determined from the corresponding relationship between membership values versus  $H$  at  $Sr_{opt}=0.74$ .

In view of the results so far achieved, the proposed multiobjective fuzzy optimum structural design method can determine the global optimum solution of large scale structures rationally, systematically and efficiently by taking into account budget limitation of construction project, relative significances of construction cost and aesthetic feeling of the structure, designer's preferences, design emphasizes of the structure and fuzziness of the decision-making.

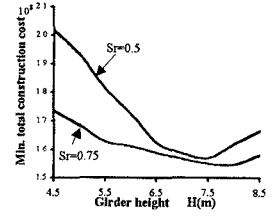


Fig. 2. Relationships between min. total construction cost and girder height at  $Sr = 0.5$  and  $0.75$

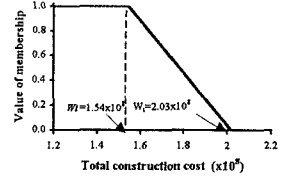


Fig. 3. A measure of membership function of total construction cost

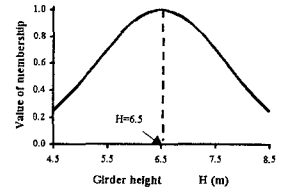


Fig. 4. A measure of membership function of aesthetic feeling

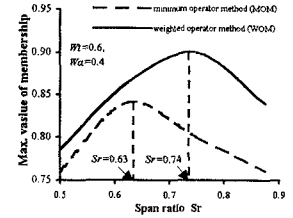


Fig. 5. Determination of the global optimum span ratios by MOM and WOM