STRUCTURAL SYSYEM IDENTIFICATION BY USING THE ADAPTIVE H_∞ FILTER

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1. Introduction

In the past several years, H_∞ control has received extensive interest and has been applied successfully into aerospace and mechanical engineering as a robust control approach. The H_∞ filter which is based on the H_∞ criterion has also been applied to the identification of linear structural system [1].

In this study, the H_∞ filter is employed to identify non-stationary parameters of linear structural system. By adding a function of memory fading for past observation data to the H_∞ filter, we developed an algorithm which can track the nonlinear property of the structural system. Digital simulation result for the identification of linear structural system with non-stationary parameters shows the effectiveness of the proposed algorithm.

2. The H_∞ Filter

Consider a system described by

$$x_{k+1} = A_k x_k + B_k \omega_k, \quad y_k = C_k x_k + D_k v_k, \quad u_k = L_k x_k \tag{1-3}$$

where x_k, y_k and u_k are the state vector, measurement and the vector to be estimated with proper dimension. The exogenous signals ω_k and v_k are the process and measurement noise, respectively. And we define $R_k := D_k D_k^T$.

The H_∞ filtering problem is to find the estimates of u_k and x_k based on the measurement set $\{y_0, \cdots, y_k\}$ such that H_∞ bound can be satisfied [2]. The H_∞ filter which achieves the above mentioned H_∞ bound is given by

$$\hat{x}_k = \bar{x}_k + K_k(y_k - C_k \bar{x}_k), \quad \bar{x}_{k+1} = A_k \hat{x}_k, \hat{x}_0 = \bar{x}_0, \quad \hat{u}_k = L_k \hat{x}_k \tag{4-7}$$

$$K_k = \bar{P}_k C_k^T R_k^{-1}, \quad \bar{P}_k = (P_k^{-1} + C_k^T R_k^{-1} C_k)^{-1} \tag{8-9}$$

where $\hat{x}_k$ and $\hat{u}_k$ are the estimate of x_k and u_k, respectively; K_k is the gain of the H_∞ filter, at time step k. $\bar{x}_{k+1}$ is the predicted value of state vector at time step k+1. P_k satisfies the Riccati difference equation

$$P_{k+1} = A_k P_k \left\{ I + (C_k^T R_k^{-1} C_k - \gamma^{-2} L_k^T L_k) P_k \right\}^{-1} A_k^T + B_k B_k^T, \quad P_0 = \Pi \tag{10}$$

where Π is a positive define matrix which represents the uncertainty of the initial state. γ is a positive constant which represents the magnitude of the penalty and satisfies $V_k := \gamma^2 I - L_k P_k (I + C_k^T R_k^{-1} C_k P_k)^{-1} L_k^T > 0$ (11), where I is identical matrix. By adding a forgetting factor λ_k to Eq. (9) as

$$\bar{P}_k = (\lambda_k P_k^{-1} + C_k^T R_k^{-1} C_k)^{-1} \tag{12}$$

we obtain the adaptive H_∞ filter which can trace the nonlinear property of the structural system.

3. Algorithm for Non-Stationary Parameter Linear System Identification

Assume only structural responses of velocity and displacement are available for the system identification. The structural parameters to be identified are the non-stationary damping coefficient matrix and stiffness matrix. The mass matrix is assumed to be given, we identify the natural frequency and damping constant of each story defined by $h_i = c_i / 2\sqrt{m_i k_i}$, $\omega_i = \sqrt{k_i / m_i}$ (i=1, n). The state vector to be identified is defined by $x = \{\cdots\ z_i\ \dot{z}_i\ h_i\ \omega_i\ \cdots\}^T$. Because the state transfer equation is expressed as a non-linear equation of x, this equation must be linearized by a proper linearization scheme [3]. Then we can get the system equation $x_{k+1} = A_k x_k + d_k$.

Key Words: Identification, The H_∞ filter , Structural system, Non-stationary parameter.

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4. Structural System Identification

The identification algorithm developed in this paper is applied to a 2 DOF linear structure with non-stationary damping coefficient. The observed responses of velocity and displacement of each stories are assumed to be given by the simulated responses of a structure with constant stiffness and linear time varying damping constants from 2% at the beginning 1% at 10 sec. El Centro NS earthquake record with scaled peak value of 50.0gal is used as the input excitation. The pink noise with 5% of the standard deviation of the structural response is added to the simulated structural responses as the measurement noise. The constant forgetting factor λ from 0.9 to 1.0 is used in order to trace the non-stationary damping of the structure. In the digital simulation, the identification results are obtained when the different value of p_0 and λ are set in order to check their effect on the system identification. The results are compared to those of Kalman filter to show the performance of the adaptive H_∞ filter in the identification of the structural system.

The results show that the identification algorithm developed in our research has good behavior to track fast variation in the system parameters. The proposed algorithm based on the adaptive H_∞ filter can converge faster than the results obtained by using the Kalman filter, as shown in Fig. 1 ($\lambda = 0.98$, $p_{0,3} = p_{0,7} = 0.0001$). If p_0 can not be selected properly, the identified value by the Kalman filter has large oscillation. But the performance of H_∞ filter can not be effected by the selection of p_0 value, as shown in Fig. 2 ($\lambda = 0.98$, $p_{0,3} = p_{0,7} = 0.001$). When the value λ is set to be smaller, the identified values can converge faster, but oscillation occurs in tracking the non-stationary structural parameter, as shown in Fig. 3 ($\lambda = 0.93$, $p_{0,3} = p_{0,7} = 0.0001$).

- - - - - - - - - Real Value ———————— the H_∞ Filter ························ the Kalman Filter

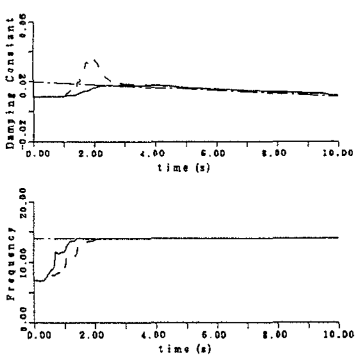
Fig.1 Identified para. of top floor

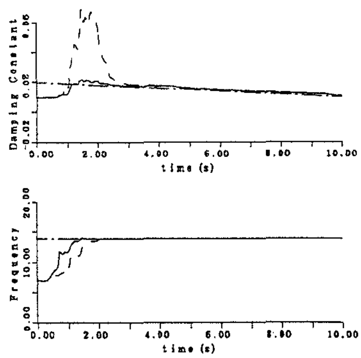
Fig.2 Identified para. of top floor

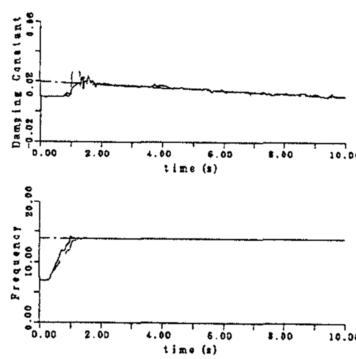
Fig.3 Identified para. of top floor

5. Conclusions

By introducing a forgetting factor to the H_∞ filter, a new structural identification algorithm was developed to be able to track the fast variation of structural parameters. The proposed algorithm was applied to a 2 DOF structural system with non-stationary damping coefficient. The digital simulation result shows that the algorithm by using the adaptive H_∞ filter can converge fast and track the non-stationary parameters effectively. Compared to the algorithm by using the Kalman filter, the proposed algorithm can keep good performance even if the initial parameters of the filter can not be set properly.

6. References

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