# V -553 Analytical Study of Energy Dissipation in Strain-Hardening Cementitious Composites

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## 1. Introduction

A new class of cement based composites reinforced by short random fibers, called engineered cementitious composites (ECCs), has been recently developed by Li [1]. The ECCs' micromechanical structure is tailored so that after cracking due to tensile or shear stress starts, normal stress transmitted by bridging fibers across the crack planes increases with increasing crack width. Such a hardening behavior leads to multiple cracking distributed over a large volume of the material. As the multiple cracks are characterized by close spacing and very small widths, the material overall behavior can be termed as pseudo strain-hardening.

The hardening relationship between bridging stress and crack opening displacement (COD) changes into softening when the COD reaches a certain critical value. This in turn triggers localization of fracture into a single crack plane.

Maalej et al.[2] experimentally measured the fracture energy of ECCs using double cantilever beam (DCB) specimens. The authors suggested that the total fracture energy $J_c$ of ECCs can be decomposed into two components. The first, denoted as $J_m$, comes from the distributed multiple cracking. The second one, called bridging fracture energy $J_b$, is associated with the fiber pull-out process at the localized fracture plane. The authors reported that ECCs exhibit high fracture energy, namely due to the extensive energy dissipation off the main localized crack plane (component $J_m$).

It appears, that due to their high fracture energy, ECCs might be efficiently used for plastic hinges in earthquake resistant structures. Note that it is desirable for such hinges to dissipate a large amount of energy. In the present paper, we investigate the energy dissipation associated with cracking of ECCs by means of finite element analysis.

## 2. Analytical model

In order to numerically study the cracking phenomena of ECCs, we use the analytical model proposed by Kabele and Horii [3]. In this model, the material in multiple cracking state is idealized as a homogenous continuum with additional strain, called cracking strain, which represents crack opening displacements a and crack spacing. Consequently, the incremental theory of plasticity is used.

The softening localized cracks are modeled as discrete discontinuities in the displacement field with the effect of fiber bridging being represented by traction applied to the crack surfaces (generalized Dugdale-Barenblatt type model). A finite element with embedded discontinuity [4] is employed to implement the above model.

## 3. Method of analysis

Double cantilever beam specimens of various sizes were used to compute the fracture energy (see Table 1). The material was an ECC consisting of cement paste reinforced with 2% by volume of Spectra polyethylene fibers. It is shown in [3] that in such specimens, first, a large round shaped zone of multiple cracking developed around the original notch tip. Later, a single localized crack with fiber bridging started to propagate from the original notch tip. As the crack mouth opening displacement increased, the contact between the localized crack surfaces was lost and a stress free crack begun to grow from the notch tip.

During the analyses, which were conducted under displacement control, the stress free (SF) crack length as well as energies dissipated by multiple cracking and on the localized crack plane were recorded for each loading increment. The energy dissipated by multiple cracking from the beginning of loading up to time $t$ was computed as the cracking strain energy:

$$E_m(t) = \int_0^t \int_V \sigma_{ij} \frac{\partial \varepsilon_{ij}^c}{\partial t} dV\, dt \qquad (1)$$

where $V$ is the specimen volume, $\sigma_{ij}$ is the stress tensor and $\varepsilon_{ij}^c$ is the cracking strain. Similarly, the energy dissipated on the localized crack plane was computed as work done by the bridging stress:

$$E_b(t) = \int_0^t \int_A \sigma_n \frac{\partial \delta_n}{\partial t} dA\, dt \qquad (2)$$

where $A$ is the localized crack surface area, $\sigma_n$ is the bridging stress (only normal component is considered) and $\delta_n$ is the normal crack opening displacement.

Because all quantities $c$, $E_m$ and $E_b$ were recorded as functions of loading time $t$, one can express the dissipated energies also as functions of SF crack length $c$. The two components of fracture energy, $J_m$ and $J_b$, are then computed as rate with respect to the SF crack length $c$, of the energy dissipated by the multiple cracking and of the energy dissipated on the localized crack plane, respectively:

$$J_m(c) = \frac{dE_m(c)}{dc} \quad \text{and} \quad J_b(c) = \frac{dE_b(c)}{dc} \qquad (3) \text{ and } (4)$$

Note that Eqs. (3) and (4) imply that, generally, the fracture energy depends on the stress free crack length.

### 4. Analytical results

The analytical results are summarized in Fig. 1 and Fig. 2. The figures show (a) fracture energy due to multiple cracking $J_m$ and (b) bridging fracture energy $J_b$, plotted against the distance between the propagating stress free crack tip and the specimen vertical surface opposite to the notch mouth. Fig. 1(a) shows that if the specimen height is fixed, $J_m$ is very much affected by the distance between the SF crack tip and the specimen surface. It is seen that in the larger specimen (w=120 cm), $J_m$ is initially almost constant but starts to drop as soon as the SF crack tip reaches the distance of about 75 cm from the specimen surface. The initial ligament of the smaller specimen (w=66 cm) is already shorter than this distance. Thus the fracture energy component $J_m$ is decreasing as soon as the SF crack starts to propagate. Fig. 1(b) shows that the bridging fracture energy is almost constant until the value of w-(c+a) becomes about 35 cm.

Table 1 Dimension of DCB specimens

| h (cm) | w (cm) | a (cm) |
|---|---|---|
| 90 | 66 | 15 |
| 90 | 120 | 15 |
| 45 | 120 | 15 |

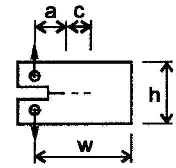


Fig. 2(a) shows that by decreasing the specimen height, the stabilized value of $J_m$ when SF crack tip is far from the vertical surface also significantly decreases. On the contrary, the component $J_b$ appears not to be affected by the specimen height at all.

The above phenomena can be explained by observing the distribution of cracking strain (multiple cracking) and the extension of the localized crack with bridging. The zone of multiple cracking expands in all directions. If it reaches either horizontal and vertical surfaces of the specimen, its further development is confined, which results in reduced rate of energy dissipation. On the contrary, the localized crack extends only in horizontal direction. Thus the component $J_b$ is not affected by the specimen height. The size of the localized crack with fiber bridging is much smaller than that of the multiple cracking zone. Hence the boundary effect on $J_b$ occurs much later than in the case of $J_m$.

It is also noticeable by comparing Fig. 1 (a) and (b) that, if the distance w-(c+a) is larger than about 45 cm, the value of $J_m$ is larger than that of $J_b$. This means that in such a case, more energy is dissipated through the distributed multiple cracking than at the localized crack plane.

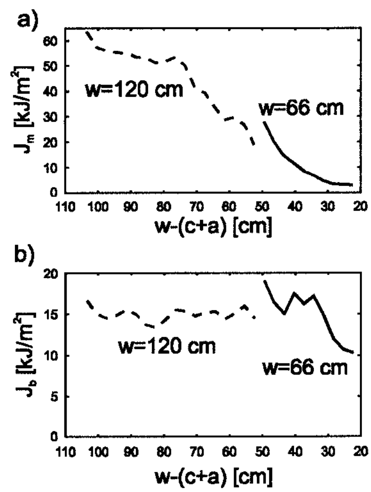


Fig. 1 Effect of specimen length w on fracture energy (h=90 cm); a) component due to multiple cracking; b) component due to localized cracking

### 5. Concluding remarks

The present paper shows that in pseudo strain-hardening composites, such as ECCs, energy is dissipated not only by propagation of a localized crack, but also by multiple cracking that takes place in a large volume of the material. Such a behavior is quite different from other, usually quasi-brittle, cementitious materials, such as concrete, where only the first mechanism significantly contributes to the fracture energy. It is also shown, that in order to utilize the ECCs' capability of energy dissipation off the main crack plane, structural members must be designed so as to allow free expansion of the multiple cracking zone.

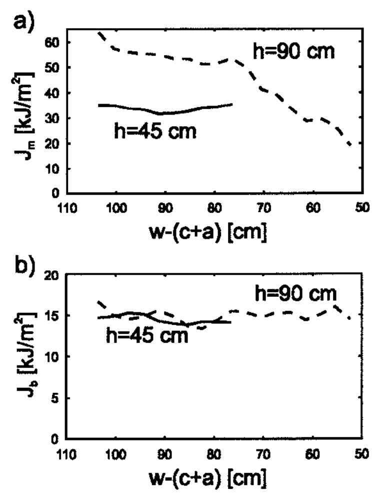


Fig. 2 Effect of specimen height h on fracture energy (w=120 cm); a) component due to multiple cracking; b) component due to localized cracking

### 6. References

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