

## A Numerical Approach for Load-Deflection Curve of Steel Fiber Reinforced Concrete

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### 1: Introduction

One of the most significant aspects of fibrous reinforcement for cement-based matrixes is the achievement of high flexural strength and toughness. Fiber reinforcement presents to the concrete properties several advantages such as superior crack control, ductility, energy absorption capacity, and improving the interior tensile strength of the concrete due to the bonding force between the fibers and the matrix [2].

Many of the field applications of steel fiber reinforced concrete as a composite material with good flexure capability are in bending rather than axial loading. Thus the flexural behavior of steel fiber reinforced concrete is the best discussed in the forms of load-deflection curves, because of the ultimate flexural strength and load-deflection diagram are the only material characteristic determined.

For understanding the flexural behavior of steel fiber reinforced concrete (SFRC) and calculate the toughness of matrix as a measure of ability to absorb energy during deformation, load-deflection curve of SFRC in the flexure is needed. For this purpose, in this study a general equation to represent the load-deflection relationship of steel fiber reinforced concrete in the flexure was proposed. The shape of the curve and the effectiveness parameters were discussed and in comparative the numerical approach and experimental results were compared to show correlation of this study.

### 2: Conditions For The Load-Deflection Relationship Equation

Two stages of flexural behavior of fiber reinforced concrete have been observed in the flexural beams. The initial stage, in a typical load-deflection relation, is linear. In this stage the composite material behaves elastically in both tension and compression zones, likewise, both the stresses and strains vary linearly across the specimen depth. Upon increasing the loading, the relation between load and deflection continues to be linear until the load reaches a point where we will refer to as the first cracking load, when the graph of this relation starts to deviate from a straight line. The transition from the linear to the nonlinear behavior is usually smooth. To explain this behavior we believe that the transition point, making the end of elastic state, occurs when the flexural stress, in tension zone of the beam, equals the cracking stress of the composite material. Upon increasing the loading a crack will be initiated; this crack starts propagating as the applied load is increased. When the matrix cracks, the stress is transferred to the steel fibers crossing the propagating crack length, while the uncracked part of the matrix still carries its share of the flexural tensile strength. As the steel fibers start to pull-out of the concrete matrix, in the L-D curve variation was occurred as a increasing the deflection and decreasing the load rate. The maximum value of the applied load will be called the ultimate flexural load and denoted by  $P_{Max}^u$  and deflection corresponding with the maximum flexural load as  $\delta_{Max}$ .

The specification of equation must be satisfied the following conditions:

- a: Ascending and descending branches should be shown.
- b: The mathematical form should be simple.
- c: The equation should compare favorably with experimental data from carefully conducted experiments.

Experimental results show that ultimate strengths of SFRC are affected by partial fiber alignment, mode of loading, size of section and span of specimen [2,4]. It was observed that by decreasing size of section with the same span or increasing in the span with the same section, the ultimate strength of SFRC were decreased. In small size specimens in the reason of partial fiber alignment increasing in the ultimate strength were observed. It is evident that, effects of high aspect ratio or increasing amounts of steel fibers in mixture increase the strength of matrix, too. Establishing a relation to show numerically the L-D curve behavior for different SFRC beams needs to affect the above factors which are effects in the curve of L-D. Those parameters are strongly dependent on test conditions, although they are not necessarily independent of each other, as will be discussed later in this paper.

### 3: Proposed Equation

After investigating several empirical expressions available in the literature, the expression proposed by Carreira and Chu [1] for stress-strain curve of plain concrete in compression were selected and modified here to investigate the load-deflection curve of steel fiber reinforced concrete in the bending. The equation should be based on physically significant parameters that can be experimentally determined and also should be only one equation for both the ascending and descending branches. The most common parameters with physical significance used to define the load-deflection relationship include  $P_{Max}^u$  as maximum flexural load and  $\delta_{Max}$  the deflection corresponding with the maximum flexural load.

$$P = \frac{P_{Max}^u \left( \frac{\delta}{\delta_{Max}} \right)^{\mu_b} \times \mu_b}{\mu_b - 1.0 + \left( \frac{\delta}{\delta_{Max}} \right)^{\mu_b}}$$

where:

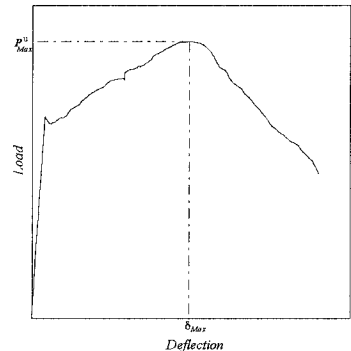


Fig. 1 : Typical Load-deflection of SFRC

$P$  : Flexural load of SFRC in general,  $kN$

$\delta$  : Center deflection in general,  $mm$

$\mu_b$  : Material coefficient

$P_{Max}^u$  : Maximum flexural load of SFRC,  $kN$

$\delta_{Max}$  : Center deflection at maximum load,  $mm$

After regression analysis in order to find the best relationship which can have good correlation with the real behavior of SFRC in the bending and can numerically calculate the load-deflection curve in the flexure the below equation was practically found have the best correlation with experiments data.

$$P = \frac{1.3P_{Max}^u \left( \frac{\delta}{\delta_{Max}} \right)}{0.3 + \left( \frac{\delta}{\delta_{Max}} \right)^{1.3}}$$

According to comparative study of the flexural behavior of SFRC beams the effects of size, span, fiber amount in the matrix and aspect ratio of steel fibers had direct relationship to the flexural load of SFRC and indirect relationship to the deflection of SFRC beams. The center-point loading mode has been shown higher flexural load and lower deflection compared to the third-point loading mode. It means, the effect of loading mode had directly effect in the amount of the flexural load and corresponding deflection and also in L-D curve.

### 3: Results

In this investigation experimental data of reference [3] were used to show the correlation relationship of the proposed equation. The size of flexural beams have been  $102 \times 102 \times 508$  mm and had been reinforced with 1.5% by volume of different fibers, which include Duoform Steel (National Standards), crimped steel (GKN) and hooked steel

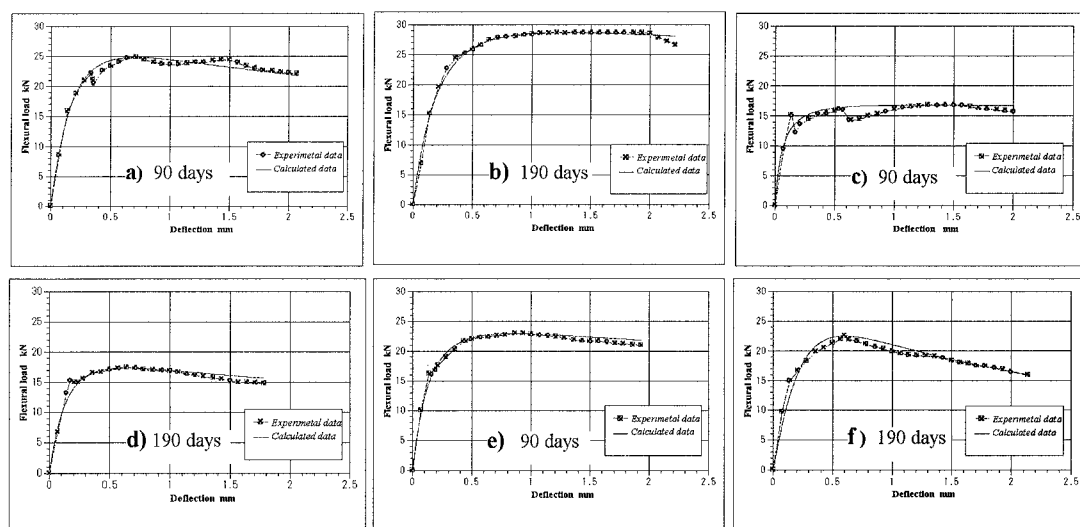


Fig. 2 : Load-deflection curve for beams reinforced with  
a & b) Duoform steel fibers, c & d) crimped steel fibers, e & f) hooked steel fibers

(Bekaert). All beams had been loaded at the third-points over 406 mm span. The results of experimental data and regression analysis of this study are shown in Fig. 2.

### 4: Conclusions

The following results from this investigation can be concluded:

- 1: The proposed equation for plotting load-deflection curve of SFRC in the flexural showed good correlation with experiments data.
- 2: Numerically approach to the load-deflection curve of SFRC beams only need maximum flexural load and corresponding deflection.
- 3: According to this equation, calculation of area under the load-deflection curve for toughness purpose become easy.

### References

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