Ⅳ-219 Explanation for Lanes' Speed Differences in a Multi-lane Freeway

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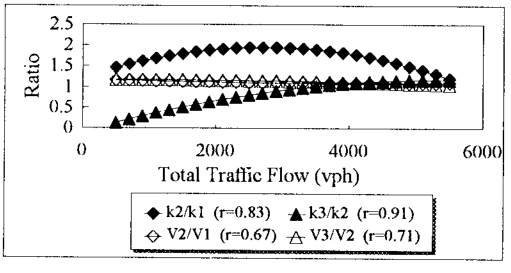
1. Introduction

Traffic distribution analysis in a three lane freeway shows[1] that the ratio of speed between adjacent lanes in uncongested condition almost having an identical linear relationship with increasing total traffic flow, as can be seen from Figure 1. Thus, in this paper, the reason for this identical relationship was examined by developed simplified theoretical concepts. Further, it is a general understanding that in order to maintain his desire speed, a diver has to optimize many constraints, such as number of overtaking, number of lane changes and acceptable gaps for these overtaking or lane changes (otherwise braking events). Moreover, when a driver has to face these constraints, his' responses can be noticed by the application of brake pedal (illuminated brake light), lane changes or overtaking and so on. Therefore, in this study, the occurrences of these constraints were considered for explaining these speed differences in individual lanes in a multi-lane freeway.



Suffix : 1 = shoulder, 2 = middle & 3 = median lane

Figure 1 Results of Ratio Analysis

2. Theoretical Concepts

In practice, there are two types of constraints for drivers such as active constraints and passive constraints. The active constraints can be expected by a follower, when he catches up with another vehicle travelling a head of him in the same lane, and the passive constraint is a result of vice versa. Let, the observed probability density function of speed *(v)* in a lane will be denoted as $f(v)$, the density of vehicle in this lane is 'k' and a proportion $f(v)dv$ of vehicle has speeds in infinitesimal range from v to $v+dv$. Further, assume an imaginary observer travel with a speed 'u' in this traffic stream. Hence, the rate at which, this imaginary observer meets the active constraints per time interval can be calculated by a simple procedure, as given by the equations (1) and (2).

$$\xi_c^a = k\int_0^u (u-v)f(v)dv \tag{1}$$

$$\xi_c^a = k\left[u\int_0^u f(v)dv - \int_0^u vf(v)dv\right] \tag{2}$$

Similarly, total number of passive constraints can be calculated as given by equations (3) and (4).

$$\xi_c^p = k\int_u^\infty (v-u)f(v)dv \tag{3}$$

$$\xi_c^p = k\left[u\int_0^u f(v)dv - u + \underline{v} - \int_0^u vf(v)dv\right] \tag{4}$$

Here, $\underline{v}$ is the average speed of the traffic stream. Hence, the total number of constraints can be written as summation of active and passive constraints, as given by equation (5).

$$\xi_c^{a+p} = k\left[2u\int_0^u f(v)dv - u + \underline{v} - 2\int_0^u f(v)dv\right] \tag{5}$$

Further, the difference between active and passive constraints can be obtained from the equations (2) and (4) as given by equation (6).

$$\xi_c^a - \xi_c^p = k(u-\underline{v}) \tag{6}$$

This equation (6) shows, if the imaginary vehicle is stationary, (i.e., $u = 0$ e.g., by accident) then all the vehicles *($k\underline{v}$)* have to face the active constraints. Further, if the imaginary vehicle travels with the average speed of the traffic stream *($u = \underline{v}$)*, then the amount of active constraints he meets during a unit of time is exactly same as the amount of passive constraints he meets. It means that, for a vehicle travelling with the average speed has to meet the same amount of active and passive constraints, which is the highest under higher flow rate condition.

Because, the usual statistics for describing the vehicles' speeds in a lane are average value and the standard deviation. Analysis shows, when the total traffic flow becoming higher, the value for standard deviation becoming smaller. i.e., more vehicles are travelling with a speed closer to the average speed. Therefore it can be said that the patterns of speed distributions between individual lanes have a strong relationship with these defined drivers' constraints.

3. Simplified Models

3.1 Simple Constraint Model

Here, a simple constraint model was used to examine the reason for the identical relationship in the ratio of speed between adjacent lane in a multi-lane freeway. It is true that, occurrences of active constraints cause a nervous outburst for drivers in an uncongested flow. This manoeuvres of driver's and traffic characteristics were examined by a simple constraints model. The following time-distance diagram was used as shown in Figure 2.

Let, in a multi-lane freeway, the 'i'th lane's traffic density is k_i, all the vehicles are travelling with an average speed of v_i and total traffic flow is q_i $(=k_i v_i)$. Further, suppose an imaginary vehicle is travelling on this lane with a speed of v_j $(>v_i)$. This Figure 2 shows, the occurrence of meeting points of vehicles' movements or in other words the occurrence of active constraints for the imaginary vehicle.

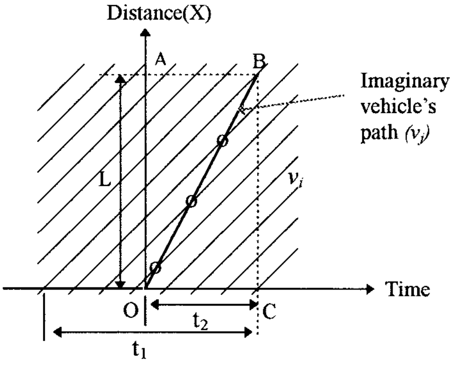


Figure 2 Vehicles' movements by x-t diagram

By a simple procedure, the expected number of active constraints per time interval $\xi^a_{c,i}$ can be obtained as given by equation (7).

$$\xi^a_{c,i} = q_i(t_1 - t_2) = k_i (v_j - v_i) \quad (7)$$

Hence, if this imaginary vehicle is travelling in this lane 'i', then the probability (ratio) of expected number of constraints for this vehicle can be obtained from dividing it by lane traffic flow rate (q_i), as given by equation (8).

$$P(\xi^a_{c,i}) = v_j/v_i - 1 \quad (8)$$

However, this expression is for an average value than a form of integral, because, it was assumed that there only a group of vehicle travelling in the lane 'i' with a speed of v_i. Therefore, another integral constraints model is necessary to describing the differences in speeds.

3.2 Integral Constraints Model

In this section, the differences in vehicles speed was considered, from the knowledge of the section 2. Hence, the expected number of active constraints can be given equation (9).

$$\xi^a_{c,i} = k_i \int_0^{u_i} (v_j - v_i) f(v_i) dv_i \quad (9)$$

Further, the total traffic flow (q_i) in the lane 'i',

$$q_i = k_i \int_0^{\infty} v_i f(v_i) dv_i \quad (10)$$

Therefore, the probability of expected active constraints can be given by equation (11).

$$P'(\xi^a_{c,i}) = \int_0^{u_i} (v_j - v_i) f(v_i) dv_i \Big/ \int_0^{\infty} v_i f(v_i) dv_i \quad (11)$$

This equation (11) also shows that, the expected probability of constraints as a function of speeds ratio. Therefore, it can be said that, this integral model also supports the above simple constraints model.

3.3 Applications and Results

By substituting $j=2$ (i.e., middle lane) and $i=1$ (i.e., shoulder lane) in equation (8), the probability of expected number of active constraints can be obtained. This result shows that the probability of expected active constraints is function of speed ratio between shoulder lane and middle lane. A similar result can be obtained by considering the middle lane and median lane. Moreover, analytical results from a three lane freeway show that, with the increasing total flow rate, the ratio of v_3/v_2 and v_2/v_1 is almost constant and overlaying relationship with one over other as shown in Figure 1. This indicates that, the drivers are balancing these numbers of expected active constraints between lanes. Therefore, it can be concluded that the pattern of existing speed distribution in an uncongested traffic flow is a result of balancing these expected drivers' active constraints between lanes in a multi-lane freeway.

4. Conclusion

In this study, the existing pattern of speed ratio between adjacent lanes in an uninterrupted segment in a multi-lane freeway was explained by a simplified theoretical concept. Developed theoretical concepts show that the defined active constraints for drivers such as number of overtaking, number of lane changes and acceptable gaps for these overtaking or lane changes (otherwise braking events) are playing an important role on this existing pattern. Analytical results in a three lane freeway show that drivers in a multi-lane are balancing these active constraints between lanes. Therefore finally, it is concluded that balancing these active constraints between adjacent lanes in a multi-lane freeway is the reason for existing pattern of the identical relationship for ratio of speed between adjacent lanes.

5. References

1. Okura. I and Somasundaraswaran. K (1996). Lane Distribution Behavior in Three Lane Uni-Directional Freeway, JSCE, Proceeding of Infrastructure Planning No. 18(1), pp. 197-200.
2. Leutzbach. W (1972). Introduction to the Theory of Traffic Flow, Springer-Verlag, Berlin Heidelberg, New York 1972.