IV－189 RANDOM DISTRIBUTION OF VALUE OF TIME IN DISCRETE CHOICE MODEL FOR ECONOMIC ANALYSIS OF TRANSPORTATION PROJECTS

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1. INTRODUCTION

Value of time (VOT) is a key element in transport planning. It influences economic evaluation of travel time savings, which is a major constituent of the benefits of a transport project, as well as the relative importance of time versus cost in travel forecasting models. In conventional methods, value of time is derived through trade-off ratio implied by time and cost coefficients of utility or generalized cost functions of the models. Here it is assumed that the trade off ratio remains same for all members or specified groups of population in both intra-personal and inter-personal domain. To incorporate inter-personal variation in the models value of time has been allowed to vary along some observed dimensions, such as income, trip purpose, mode of travel etc. This kind of model is known as random coefficient model and has been used extensively in VOT studies in United Kingdom and the Netherlands [Ben Akiva, 1993].

In actual choice situation, the relative importance of time and cost, i.e. value of time, may vary among individuals. Even within an individual, it may be influenced by tastes and circumstances. These variations can not be observed. Although one can not model such factors explicitly, it may still beneficial to try to identify the influence of these factors on estimated value of time. This may enhance the confidence on the use of the estimated value in economic analysis and pricing decisions. A method for incorporating distribution of value of time in choice model is derived in this paper. The results of a case study of application of the method are also presented.

In the paper section 2 explains the theory and formulation of the model, section 3 describes the data used for empirical analysis and section 4 presents the results of the analysis. Section 5 explains the implications of the results and, also provides concluding remarks as well as further research topics on the issue.

2. THEORY AND FORMULATION OF THE MODEL

2.1 Detail Specification of Model Structure

The generalized cost, for the n-th individual choosing i-th alternative, is assumed to be a random variable given by,

$$GC_i^n = x^n t_i^n + c_i^n + \xi_i^n \qquad \textit{Where, error term is} \quad \xi_i^n \to N(0,\delta^2) \qquad (i)$$

Using the principle of minimization of generalized cost, the probability of choosing alternative i over alternative j is given by,

$$\Pr(i) = \Pr(GC_i^n \le GC_j^n)$$

2.2 Distribution of Value of Time in the Model

All the available literature postulate that value of time is log-normally distributed. But this assumption makes the incorporation of stochastic behavior in the model extremely difficult and, the model becomes mathematically intractable. Also, while analyzing the data we have observed that the trade-off between time and cost [as defined as $x^*=(c_i-c_j)/(t_j-t_i)$] is not always positive [Alam et. al., 1995]. To make the model computationally tractable and to include practically feasible negative values of x^* in the analysis, it is assumed that value of time is normally distributed. Given the mean x_{mean} and standard deviation σ, the distribution of VOT can be shown as,

$$f(x^n | x_{mean}^n, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{1}{2\sigma^2}\left(x^n - x_{mean}^n\right)^2\right\}$$

2.3 Formulation of the Model

The value of time can be written as, $x^n = x_{mean}^n + \upsilon^n$ $\qquad$ *Where* $\upsilon^n \to N(0,\sigma^2)$

So the generalized cost can be expressed as, $GC_i^n = x_{mean}^n t_i^n + c_i^n + \upsilon^n t_i^n + \xi_i^n$ $\qquad (ii)$

Let $\varepsilon_i^n = \upsilon^n t_i^n + \xi_i^n \to N(0, \sigma^2 t_i^{n2} + \delta^2)$, the generalized cost is given by $GC_i^n = x_{mean}^n t_i^n + c_i^n + \varepsilon_i^n$

The probability of choosing alternative i instead of j is given by,

$$\Pr(i) = \Pr(GC_i^n \le GC_j^n) = \Pr(x_{mean}^n t_i^n + c_i^n + \varepsilon_i^n \le x_{mean} t_j^n + c_j^n + \varepsilon_j^n)$$

$$\Pr(i) = \Pr\left\{\varepsilon_i^n - \varepsilon_j^n \le x_{mean}^n (t_j^n - t_i^n) + (c_j^n - c_i^n)\right\}$$

Each of the random terms, ε_i^n *and* ε_j^n is composed of two components. One is pure noise term, which can be assumed to be independent of each other. The other component is related to the distribution of value of time. The distribution of value of time for the choice alternatives within an individual can not be assumed to be independent. This implies that the terms should not assumed to be completely independent. Their degree of correlation depends on the relative value of the components. Assuming that the correlation coefficient is ρ, the variance of $\varepsilon^n (= \varepsilon_i^n - \varepsilon_j^n)$ is given by,

$$VAR = 2\delta^2 + (t_i^{n2} + t_j^{n2})\sigma^2 - 2\rho\sqrt{(t_i^{n2}\sigma^2 + \delta^2)(t_j^{n2}\sigma^2 + \delta^2)}$$

And corresponding choice probability is given by,

$$\Pr{}^n(i) = \Phi\left\{\frac{x_{mean}^n (t_j^n - t_i^n) + (c_j^n - c_i^n)}{\sqrt{2\delta^2 + (t_i^{n2} + t_j^{n2})\sigma^2 - 2\rho\sqrt{(t_i^{n2}\sigma^2 + \delta^2)(t_j^{n2}\sigma^2 + \delta^2)}}}\right\} \qquad (iii)$$

It is not possible to estimate the parameters of this expression using available mathematical techniques and softwares. But the model can be simplified to analyze value of time in two extreme situations.

A. If the data is relatively free of noise, the two random variables become perfectly correlated. We can assume that, $\delta \approx 0$ *and* $\rho \approx 1$. The choice probability is given by,

$$\Pr{}^n(i) = \Phi\left\{\frac{x^n_{mean}(t^n_j - t^n_i) + (c^n_j - c^n_i)}{\sqrt{(t^{n2}_i + t^{n2}_j)\sigma^2 - 2t^n_i t^n_j \sigma^2}}\right\} = \Phi\left\{\frac{x^n_{mean}(t^n_j - t^n_i) + (c^n_j - c^n_i)}{\sigma\left(\left|t^n_j - t^n_i\right|\right)}\right\} \qquad (iv)$$

B. If the noise terms dominate, the two random variables become independent. We can assume that, $\sigma \approx 0$ *and* $\rho \approx 0$. The resulting model becomes similar to probit model. The choice probability is given by,

$$\Pr{}^n(i) = \Phi\left\{\frac{x^n_{mean}(t^n_j - t^n_i) + (c^n_j - c^n_i)}{\sqrt{2\delta^2}}\right\} \qquad (v)$$

3. BRIEF DESCRIPTION OF DATA

The data used to implement the approaches explained above was collected in a mail-back Stated Preference (SP) survey in Tokyo metropolitan area at the end of 1994. A detailed description of the data and SP survey has been reported by S. Zhao et al (1995). The sample was composed of 120 car commuters who used toll expressways in Tokyo. The questionnaire contained several questions about socioeconomic variables of the respondent together with a number of SP questions. For this analysis, two experiments have been used for each respondent.

4. RESULTS OF THE ANALYSIS

The probability expressions given by equation *(iv)* and *(v)* have been analyzed using Maximum Likelihood method in GAUSS environment. Table 1 provide the results of the analysis of noise free and noise dominant cases.

Table 1: Estimated Parameters

	SP Survey for Choice Between Surface Road and Toll Expressway	
Variables	Model (iv) Low Noise Case	Model (v) Noise Dominant Case
Mean Value of time Yen/min	66.86 (5.65)	34.34 (7.91)
Standard Deviation of VOT, σ Yen/mi	93.40 (5.43)	-
Standard Deviation of Noise, δ Yen	-	665.33 (8.37)
$L(pr = 0.5)\text{x}10^3$	-166.36	-166.36
$L(\beta_e)\text{x}10^3$	-114.61	-101.65
ρ^2	0.31	0.39
Number of Observations	120	120

( ) t-Statistics, $L(pr = 0.5)$ initial Log-Likelihood assuming all probability to be 0.5, β_e estimated parameters

The mean value of time varies from 34.34 to 66.86 yen per minute. As expected, the result of noise dominant case is consistent with the results of ordinary logit or probit analysis. The estimated standard deviation of VOT implies that the upper limit of the coefficient of variation of VOT is about 1.40. The noise variance is also very high, much higher than the variance of VOT. The goodness of fit statistics are quite interesting as both of them are statistically satisfactory. The model which considers the noise term provides slightly better statistics. The *t*-statistics of all the estimated parameters of both the models are satisfactory. These observations support the theoretical structures of the models and implies that when estimated simultaneously, the mean value of time, its standard deviation and standard deviation of noise should lie within the range provided by the models with higher goodness of fit statistics.

5. CONCLUSION

A method for estimating distribution of VOT as well as its implementation have been described in the paper. The overall results imply that models should not ignore either distribution of VOT (as usually done in conventional methods) or the noise term in the analysis. The results presented here provide information about the upper limits of distribution of value of time and noise terms and, will help in determining value of time more judiciously. There are a number of ways in which the approach described in the paper can be extended. Although all the parameters could not be estimated simultaneously because estimation techniques for the choice probability expression which considers all of them simultaneously have not been available, their joint effect can be visualized by simulation study. To estimate further variation of value of time a semi-parametric discrete “mass point” distribution can be incorporated. The method can also be applied to variables other than value of time to incorporate their distribution in the model. This can be easily done for the variables which are not correlated with time. The analysis of the correlated variables are not computationally feasible at present.

6. REFERENCES

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