I－B 83

WALKING INDUCED VIBRATION ON FOOTBRIDGES

Nguyen Nguyen MINH, Student member,
Pennung WARNITCHAI, Toshio MIYATA, Hitoshi YAMADA,
Member JSCE, Yokohama National University

1. Introduction

One major serviceability problem in many modern footbridges is the excessive vibrations which can annoy or even alarm pedestrians. The vibrations are basically caused by the resonance of the fundamental mode of the bridges with the periodic loads produced by walking pedestrians. Until recently, all works on this problem have performed the dynamic analysis of footbridges subjected walking loads by numerical direct integration method. This is obviously a disadvantage for designers.

In this study, an analytical closed-form solution of the resonant response of a damped footbridge under one-pedestrian walking load is derived by an approximation technique. The solution agrees very well with the numerical results obtained from direct integration method. These findings are very useful for establishing acceptance criteria and better understanding the problem since there has been no such an analytical solution so far.

2. Models of Footbridges and Walking Loads

A one-span footbridge with uniform section is considered. The structure is therefore modeled as an uniform simply supported beam with uniformly distributed properties of mass $\overline{m}$, bending rigidity EI and structural damping ratio ξ. Walking load can be characterized by the pacing rate (f_m) ranging between 1.5 to 2.4 Hz, the forward speed (V) and the load-time function. The one-pedestrian walking load is modeled as a moving pulsating load $F(t)$ moving with constant speed V along the beam.

$$F(t) = F_0 \sum_{n=1}^{N} \alpha_n \sin(2n\pi f_m t - \varphi_n) \tag{1}$$

Where: F_0 is static weight of a pedestrian, α_n, φ_n are the n^{th} harmonic coefficient and phase angle. In most cases, walking induced vibration is governed by the first harmonic component.

3. Dynamic Analysis

3.1 *Analytical Derivation of Resonant Response*

In this analysis, two simplifications are needed: (1) Only the first structural vibration mode of fundamental frequency f_s is considered; (2) Walking load is idealized as a moving sinusoidal force. It means only the first harmonic component of $F(t)$. is considered. Applying modal analysis, the response of structure can be analytically derived, but it is so lengthy and complicated. Moreover, resonant response is the value of interest. Therefore, an approximation technique is employed to simplify the analytical solution of the resonant response.

In our consideration, typical ranges of structural damping ratio (ξ) and the number of steps (n_s) needed to pass the bridge are

$$\xi \leq 0.05 \text{ and } 10 \leq n_s \leq 60 \tag{2}$$

where $n_s = L/s_t$, L is the bridge's length and s_t is walking stride length depending on f_m.

Then, there are two parameters which are very small compared to unit 1. They are

$$\varepsilon = 1/n_s \ll 1 \text{ and } \xi \ll 1 \text{ (order of } 10^{-2}) \tag{3}$$

By omitting the high order terms of these parameters in the analytical solution, the approximate closed-form of resonant acceleration response at mid-span of the bridge is obtained as

$$w(t) = -\frac{2F_0}{\overline{m}L}\frac{n_s \cos 2\pi f_m t}{(1+4\xi^2 n_s^2)}\left[\cos\frac{\pi V t}{L} - 2\xi n_s \sin\frac{\pi V t}{L} - \exp(-2\xi n_s \frac{\pi V t}{L})\right] \tag{4}$$

A typical resonant acceleration response is shown on Fig. 1. The envelope of the response is obtained by omitting the factor $\cos 2\pi f_m t$ in Eq.(4). From this envelope, the time t_{max} at which maximum response occurs can be found as

$$t_{\max} = \frac{L(\pi - \tan^{-1}(2\xi n_s))}{\pi V} \tag{5}$$

Therefore, maximum resonant response at mid span is

$$W_{\max} = (2\pi f_s)^2 \overline{U}_{st} D(n_s, \xi) \tag{6}$$

where, $\overline{U}_{st}$: static deflection of the bridge due to F_0 applying at mid span

$$\overline{U}_{st} = \frac{F_0 L^3}{48EI} \qquad (7)$$

D: dynamic amplification factor, which is a function of n_S and ξ.

$$D = \frac{96 n_s}{\pi^4 \sqrt{1+4\xi^2 n_s^2}} \left[1 + \frac{\exp\left[-2\xi n_s (\pi - \tan^{-1}(-2\xi n_s))\right]}{\sqrt{1+4\xi^2 n_s^2}} \right] \qquad (8)$$

Compared to the dynamic amplification factor of harmonic excitation case, it can be pointed out that the number of steps n_S represents the transient nature of walking load.

3.2 *Numerical Integration*:

Modal superposition analysis with five modes of structural vibration is applied. Three harmonic components of walking load are considered. Newmark's Direct Integration method ($\alpha = 0.25$ $\delta = 0.5$) is used with time step size Δt=0.01sec.

Fig.2 shows the Dynamic Amplification Factor D versus number of steps n_S at different structural damping ratio ξ by both approaches. The error between two approaches is negligibly small as shown in Fig. 3.

4. Conclusion

The dynamic analysis of a uniform simply supported footbridge under one-pedestrian walking load was performed by both analytical and numerical approaches. The results agree very well to each other. Therefore, the newly developed analytical closed-form of resonant response gives a better understanding on the problem, provides a good basis to explain results from other works and to develop the acceptance criteria.

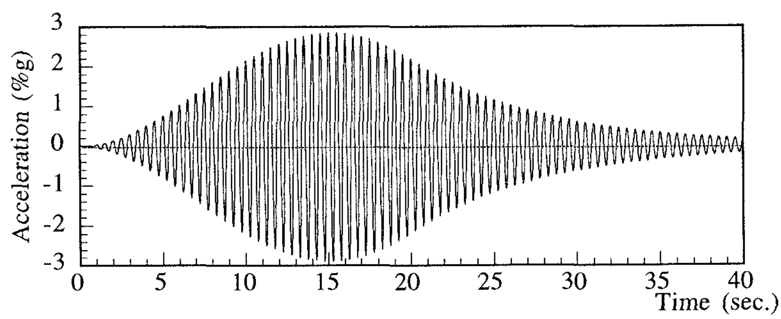

Fig. 1: Time History of Acceleration Response at Mid span of the Bridge.

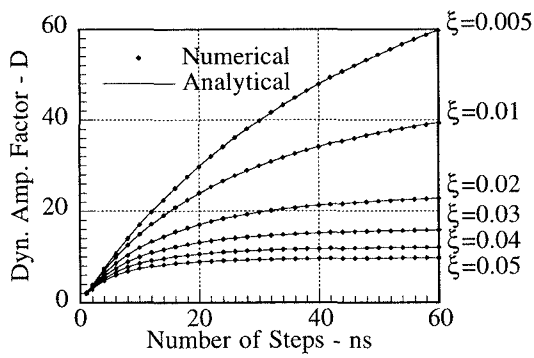

Fig.2: Dynamic Amplification Factor D(ns,ξ)

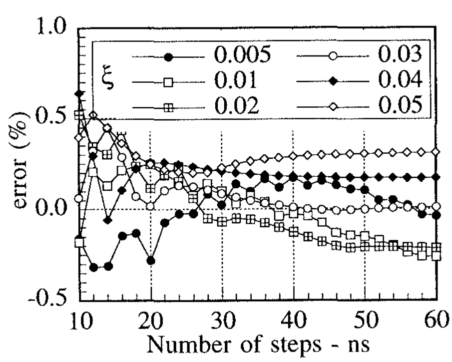

Fig.3: Error between Analytical & Numerical Approach at Different Damping Ratio ξ

5. References

1. BACHMANN, H. and AMMANN, W., *Vibration of Structures Induced by Man and Machine*, IABSE-AIIPC-IVBH, Structural Engineering Documents, 3e, 1987.

2. Nguyen Nguyen MINH, 'Man Induced Vibration on Pedestrian Bridges', *Master of Engineering Thesis*, School of Civil Engineering, Asian Institute of Technology, 1995.