土木学会第51回年次学術講演会（平成８年９月）

I－B３

Active Control of Large Structures under Seismic Loads

Yokohama National University — Paulet-Crainiceanu Fideliu — Student Member
Yokohama National University — Miyata Toshio — Member
Yokohama National University — Yamada Hitoshi — Member

INTRODUCTION

Recent advances in active structural control rose the hope that huge investments, like long span cable bridges and tall buildings, and their users to be efficiently protected from natural actions, like typhoons and strong earthquakes. Intensive studies and some practical applications were conducted for buildings but only few for bridge structures.

The most efficient implementation of active control is providing it from design stage thus leading to a lower cost for construction. However, from reasons of safety and lack of experience, the decision of active control adoption is taken after design and/or after completion. A such decision could be drawn based on some facts: i) unexpected high seismic activity in some areas may lead to modifications in design intensity level associated to those areas; ii) after strong earth shakings, the design procedures could suffer dramatic changes; iii) existing bridges, after a period of service, might present damages; iv) retrofit of existing bridges which had shown poor behavior during past seismic events; v) assurance of “damage free” response of bridges' structures to earthquake loading; vi) retrofit of existing bridges to which the dead or live loads conditions suffered changes and wrisk high damages to seismic action; vii) for bridges with traffic that cannot be stopped or could generate high losses if stopped during strong earthquakes; viii) when passive control does not give enough protection against earthquake actions.

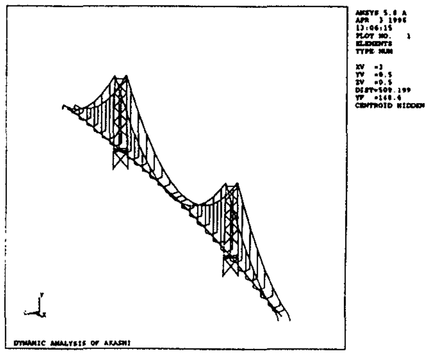

Figure 1

OPTIMAL CONTROL STRATEGY

The optimal active control is a time domain strategy which is appropriate for controlling the response of structures to earthquakes. The strategy allows to minimize the energy induced in structure, which is the aim of the seismic design.

The equation of motion for a *n* degree of freedom controlled system under seismic action is:

$$\mathbf{M}_1\ddot{\mathbf{z}}(t)+\mathbf{C}_1\dot{\mathbf{z}}(t)+\mathbf{K}_1\mathbf{z}(t)=\mathbf{f}(t)+\mathbf{u}(t) \qquad (1)$$

where: $\mathbf{M}_1$ is the $n\times n$ mass matrix of the structure, $\mathbf{C}_1$ is the $n\times n$ damping matrix; $\mathbf{K}_1$ is the $n\times n$ stiffness matrix, $\mathbf{z}(t)$ is the *n* dimensional vector of generalized displacements, $\mathbf{u}(t)$ is the *n* dimensional vector of control actions. $\mathbf{f}(t)$ is the *n* dimensional vector of external actions and is taken $\mathbf{f}(t)=\mathbf{h}_1\ddot{x}_g(t)$ where $\mathbf{h}_1$ is a *n* vector showing the points of application and the values for inertia and $\ddot{x}_g(t)$ is the ground seismic acceleration. $\mathbf{h}_1=-\mathbf{M}_1\mathbf{l}$ where $\mathbf{l}$ is a *n* dimensional vector with ones for those degrees of freedom affected by inertia and zeros for the rest.

Then equation (1) can be written as a state equation:

$$\dot{\mathbf{x}}(t)=\mathbf{A}\mathbf{x}(t)+\mathbf{B}\mathbf{u}(t)+\mathbf{h}\ddot{x}_g(t) \qquad (2)$$

with the next notations:

$$\mathbf{x}=\begin{Bmatrix}\mathbf{z}\\ \dot{\mathbf{z}}\end{Bmatrix},\quad \mathbf{A}=\begin{bmatrix}\mathbf{0} & \mathbf{I}\\ -\mathbf{M}_s^{-1}\mathbf{K}_s & -\mathbf{M}_s^{-1}\mathbf{C}_s\end{bmatrix},\quad \mathbf{B}=\begin{bmatrix}\mathbf{0}\\ \mathbf{M}_s^{-1}\end{bmatrix},\quad \mathbf{h}=\mathbf{B}\mathbf{h}_1 \qquad (3)$$

The system of *n* second order differential equations (1) became a *2n* first order differential equations (2) and so that **A, B** and **h** are $2n\times 2n$, $2n\times n$ and *n* respectively.

Supposing that the control actions are a function of the states, i.e $\mathbf{u}(t)=-\mathbf{K}(t)\mathbf{x}(t)$ then the goal is to obtain the feedback gain matrix $\mathbf{K}(t)$ such that to minimize a performance index *J* defined by

$$J = \int_0^{t_f} \left[\mathbf{x}'(t)\mathbf{Q}\mathbf{x}(t) + \mathbf{u}'(t)\mathbf{R}\mathbf{u}(t) \right] dt \qquad (4)$$

where the matrices **Q** and **R** are weighting matrices and the prime sign means the transpose. **Q** and **R** show the relative importance of the states and of the actuating forces.

After some manipulations, and knowing that the Riccati matrix $\mathbf{P}(t)$ involved in solving the problem is usually a constant matrix with sharp decrease to zero in the vicinity of the end of the control time interval, t_f, one can get the next Riccati time independent matrix equation:

$$\mathbf{PA} - \frac{1}{2}\mathbf{PBR}^{-1}\mathbf{B}'\mathbf{P} + \mathbf{A}'\mathbf{P} + 2\mathbf{Q} = \mathbf{0} \qquad (5)$$

and the control matrix gain is a constant matrix:

$$\mathbf{K} = \frac{1}{2}\mathbf{R}^{-1}\mathbf{B}'\mathbf{P} \qquad (6)$$

In order to take into account the acceleration of the structure (acceleration feedback) a different index to be minimized should be employed:

$$J = \int_0^{t_f} \left[\mathbf{x}'(t)\mathbf{Q}\mathbf{x}(t) + \dot{\mathbf{x}}'(t)\mathbf{Q}_1\dot{\mathbf{x}}(t) + \mathbf{u}'(t)\mathbf{R}\mathbf{u}(t) \right] dt \qquad (7)$$

where the new $2n \times 2n$ weighting matrix $\mathbf{Q}_1$ may be defined

$$\mathbf{Q}_1 = \begin{bmatrix} 0 & 0 \\ 0 & \mathbf{Q}_a \end{bmatrix} \qquad (8)$$

and $\mathbf{Q}_a$ is $n \times n$ weighting matrix for accelerations. If in addition we partitionate the matrices **A** and **B** such as

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_s \\ \mathbf{A}_i \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \mathbf{B}_s \\ \mathbf{B}_i \end{bmatrix} \qquad (9)$$

with $\mathbf{A}_s$, $\mathbf{A}_i$, $\mathbf{B}_s$ and $\mathbf{B}_i$ having n lines each, we can define a new Riccati problem ($\mathbf{A}^0$, $\mathbf{B}^0$, $\mathbf{Q}^0$, $\mathbf{R}^0$) instead of the initial one (**A**, **B**, **Q**, **R**) where:

$$\mathbf{A}^0 = \mathbf{A} - \mathbf{B}\mathbf{T}_{22}^{-1}\mathbf{T}_{12}', \quad \mathbf{B}^0 = \mathbf{B},$$
$$\mathbf{Q}^0 = \mathbf{T}_{11} + -\mathbf{T}_{12}\mathbf{T}_{22}^{-1}\mathbf{T}_{12}', \quad \mathbf{R}^0 = \mathbf{T}_{22} \qquad (10)$$

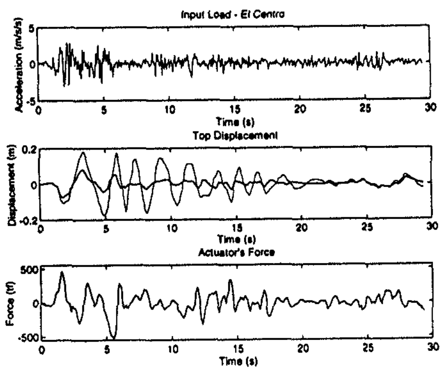

Figure 2

and

$$\mathbf{T}_{11} = \mathbf{Q} + \mathbf{A}_i'\mathbf{Q}_a\mathbf{A}_i, \quad \mathbf{T}_{12} = \mathbf{A}_i'\mathbf{Q}_a\mathbf{B}_i, \quad \mathbf{T}_{22} = \mathbf{B}_i'\mathbf{Q}_a\mathbf{B}_i + \mathbf{R} \qquad (11)$$

APPLICATION

To study the application of active control to large structures like long span bridges, see Figure 1, implies the use of large analytical models. Therefore the matrices involved in equation (1) are also large. They are composed from the structures characteristics to which are added the dynamic characteristics of the active devices, like ATMDs. It is worth to be noted that in equation (2) the dynamics of the actuators could be included, if it is known. Once the system's equation is established, the known analysis of structures can be applied. Figure 2 shows a typical time-history response of the towers of Akashi Kaikyo Bridge, under the El Centro earthquake, with and without the active control applied using ATMDs on their tops.

CONCLUSIONS

Active control of the response of large structures under seismic loads is shown to be effective by analytical approaches. Practical considerations, like actuators dynamics, time delay, power consumption, noise in measurements, failures in energy supply or actuators, nonlinearities, etc are under consideration.

REFERENCES

1. Meirovitch, L. - "Dynamics and Control of Structures", John Wiley & Sons, New York, 1990
2. Miyata, T., Kitagawa, M. - "Design Considerations for Superstructures of the Akashi Kaikyo Bridge", International Seminar on Utilization of Large Boundary Layer Wind Tunnel, Tsukuba, Japan, 1993.
3. Negoita, Al., coord. - "Inginerie Seismica" (*Earthquake Engineering*), Editura Didactica si Pedagogica, Bucuresti, 1985 (in Romanian)
4. Soong, T.T. - "Active Structural Control", Longman Scientific & Technical, 1990