I-B2 FLUTTER CONTROL OF LONG SPAN BRIDGES BY MOVING MASS

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1. INTRODUCTION

For the design of very long span bridges, wind induced vibrations are critical . Among them the most important is flutter since it can lead to a total collapse of the bridge. Passive ways to protect bridges against flutter, stiffening the deck or modifications of its aerodynamic characteristics, provide satisfactory performance for span up to 2500m. But as we are designing spans of more than 2500m, those passive solution are not enough and active solutions to the flutter problem are currently under investigation. In this study, the suppression of the flutter is by placing a moving mass inside the deck. The variation of the position of the mass creates eccentricity and provides a stabilizing moment.

2. MODELING AND CONTROL LAW

A very long span bridge is modeled by two-degree of freedom, heaving x_h and pitching x_α,as presented in Fig. 1. The motion of the moving mass is defined as a third degree of freedom, x_c. The relationship between the control signal u(t) and the displacement x_c of the moving mass is modeled by a second order differential equation with a high natural frequency, and large damping.

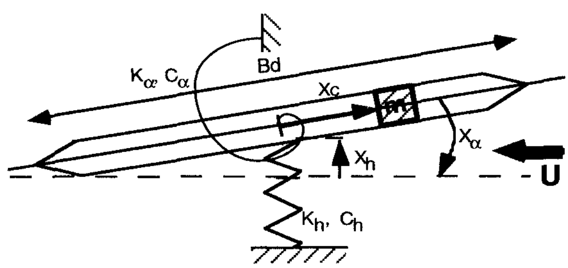

Fig. 1 Bridge Deck Model with Moving Mass

The equation of motion of the system is given by:

$$M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = F_a(t) + Bu(t), \qquad (1)$$

where:

$$x = \begin{bmatrix} x_h \\ x_\alpha \\ x_c \end{bmatrix}, \quad M = \begin{bmatrix} m_h & 0 & 0 \\ 0 & I_\alpha + x_c^2 \times m_c & 0 \\ 0 & 0 & m_c \end{bmatrix}, \quad K = \begin{bmatrix} \omega_h^2 m_h & 0 & 0 \\ 0 & \omega_\alpha^2 I_\alpha & -m_c g \\ 0 & -m_c g & \omega_c^2 m_c \end{bmatrix}.$$

The aerodynamic force vector, $F_a(t)$, consists of lift and moment which are estimated through Theodorsen's function. The time domain realization of unsteady aerodynamics was carried out using the Rational Function Approximation[1)]. The variable part of the moment of inertia was not considered in the following simulations.

Since the stroke of the moving mass is bounded by the width of the deck, a nonlinear control was selected. Applied linear-saturated control algorithm[2)] aims at minimizing the performance function J given by:

$$J = \int_0^\infty \left[x^T Q x + 2(x^T P_0 S P_0 x)^{1/2} \right] dt. \qquad (2)$$

Where Q is the weight matrix, and S and P_0 are solution of the Riccati and Lyapunov equations, respectively. The magnitude of the control force $u(t)$ is bounded by a which correspond to the maximum displacement of the mass. The control algorithm is given by:

$$u = \begin{cases} -B^T P_0 x, & |B^T P_0 x| < a \\ -sign(B^T P_0 x)a, & |B^T P_0 x| > a. \end{cases} \qquad (3)$$

The equation of motion (1) with a control (2) represent a nonlinear system.

4. EFFICIENCY OF THE CONTROL

The efficiency of the control is determined by the increase of the wind speed for which the system is stable. The stability is estimated by the describing function method presented in[3)]. The nonlinear component of the system, namely $u(t)$, is linearized and described by amplitude dependent function $N(\mathrm{A})$. The transfer function of the closed loop system is

$$T(s)=\frac{G(s)}{1+B^TP_0G(s)N(A)}. \qquad (4)$$

Where G(s) is the transfer function for the linear part of the system. Thus, the analysis of the denominator of (4) provides the stability condition and an easy method for limit cycle prediction.

The bridge used for simulations is assumed to have a main span of 4000m with the following characteristics: Bd = 40m (width of the deck), m_h= 8×10^3 kg-f/m, I_α=1×10^6 kgf-m^3/s^2, ω_h = 0.3 rad/s, ω_α=0.7 rad/s. The critical flutter wind speed for the bridge without control is 42 m/s. The mass ratio ($\mu = m_c / m_h$) was varied between 1% and 10%. Fig. 2 shows results of the control for different mass ratios, when the full width of the bridge is used i.e., the stroke is 20m. Fig. 3 shows the results when the maximum stroke is reduced from 20m to 5m, for different mass ratio (3%, 6%, 9%).

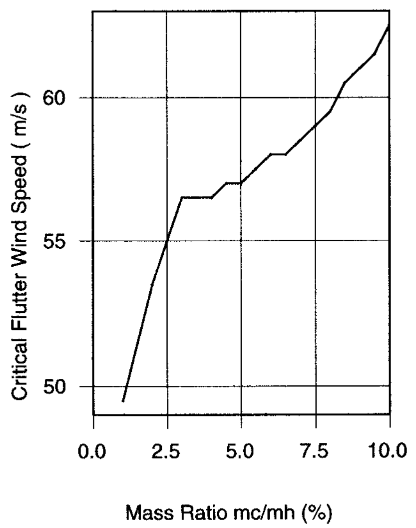

Fig.2 Critical Wind Speed for Different Mass Ratios.

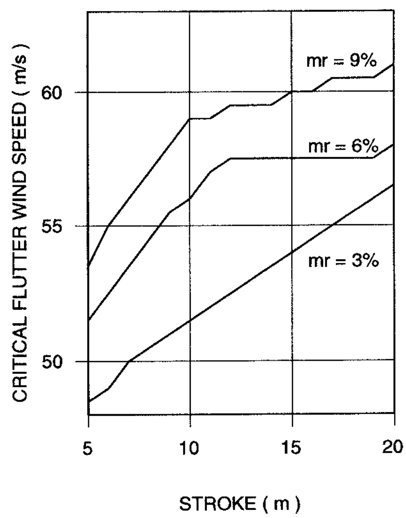

Fig.3 Critical Wind Speed for Various Maximum Strokes

For wind speed above 42 m/s, the analysis of the controlled system by describing function showed the existence of an unstable limit cycle.

4. CONCLUSION

The control used in this study proved to be effective in increasing the critical flutter wind speed for the bridge. The increase of mass ratio up to 3% (Fig. 2) provides a fast improvement of critical wind speed. In this range the gains of the control are designed primarily to suppress torsional displacement. For larger mass ratios, it is also necessary to include the suppression of vertical displacement in the gain design, therefore, the increase of flutter wind speed is smaller. The limitation of the maximum stroke of the moving mass (Fig. 3) for the small mass ratios, shows a linear decrease of the critical wind speed. For larger mass ratios, and the maximum stroke of more then 12m, the increase of the efficiency of the control is small. The capacity of the actuator and limitations of additional dead load on the bridge will determine the optimal mass ratio and maximum stroke, which is intended to be further investigated.

REFERENCES

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