

I-A 221

FLUTTER ANALYSIS BY THE MODE TRACING METHOD

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1. Introduction

The design of suspension bridge with ever longer span length has urged special attention to the prevention of flutter occurrence. Such analysis would involve a very large system with the possibility of many close modes becoming unstable. Furthermore, the mechanism of flutter is known to be closely related to the shape of vibration which can dictate the net interchange of energy between structure and flow. For low frequency, long-span bridges with inclined, cross hangers, mono-duo cable or with the active control built-in, the 3D behavior of vibration modes should be very complex. The prediction of flutter behavior of such system would require more refined analytical method which can work on the full model and at the same time computationally efficient. Method of flutter prediction based on the structural modal analysis seems to be inadequate meanwhile the method of direct flutter analysis by assuming the reduced frequency k does not give a clear picture. In this study, an iterative method based on the mode shapes of vibration is proposed. By starting from the zero wind condition, the method is like tracing the vibration mode in the complex plane.

2. Iteration Method By Tracing the Vibration Mode

The flutter behavior can be analyzed by solving the equation system (1), where \mathbf{M}, \mathbf{K} are mass and stiffness, ω is natural frequency, b is the half chord of the deck section, U is the mean wind speed.

$$\det[\mathbf{M}_F(k) \cdot \omega^2 + \mathbf{K}] = 0 \quad ; \quad k = \frac{\omega b}{U} \quad (1a, b)$$

It should be noticed that the first equation is the eigenvalue problem with the matrix \mathbf{M} parameterized by k [1]. The solution of the two equations 1a,b with 3 unknowns, U, k, ω will give the relational functions between these parameters. The flutter behavior can be best analyzed by the relation between U and the system damping. The proposed method of iteration is based on a fair assumption that the dynamic characteristic of the system does not change radically with a small increase of the wind speed. Thus, when the complex frequency and the shape of any mode at certain U are known, their corresponding values at the next step of U can be easily determined by the iteration. This calculation can be carried out by successively solving the two equations 1a,b until k, ω are converged. Graphs of equation. (1a,b) are shown in the Fig. 1 for different wind speed. The solutions are the intersection points of these two graphs which could be calculated by looping between k and ω . Inside each iteration loop, the problem of eigenvalue has to be solved. As the approximations of the eigenvectors and eigenvalues are known, a combination of the inverse iteration and the power method is used as:

$$\left[\mathbf{M}_F + \frac{1}{(\omega_i^2 - \mu)} (\mathbf{K} + \mu \mathbf{M}_F) \right] \mathbf{V} = \frac{1}{(\omega_i^2 - \mu)} \mathbf{V} \quad (2)$$

By using a eigen space shifting factor μ , any eigenvalue could be made dominant and the eigenvector \mathbf{V} will rapidly converge to the intended mode. The factor μ adjusted after each loop will close on ω^2 making the convergency rate very fast. At each iteration, only one mode is usually traced, however, to deal with the closeness in frequency, several modes can be iterated and the tracing direction will be chosen by the mode shape comparison. The calculation, starting from $U=0$ is carried out by gradually increasing U and tracing the variation of the system pole ω_i in the complex plane (Fig. 1b). By tracing single mode at once, the method proves to be accurate and computationally efficient.

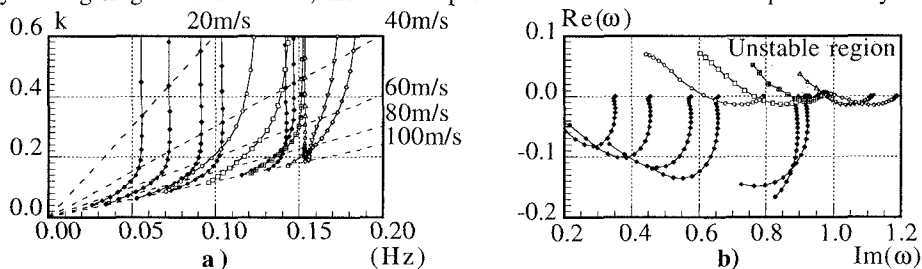


Fig. 1 The variation of frequency with k a) and the system poles in the complex plane b).

3. Numerical Example of Application

Example of 2500m span length, suspension bridge model (Fig. 2) is analyzed. The deck is assumed to be a flat, streamlined box girder where the aerodynamic forces can be calculated by the Theodorsen functions. The matrices \mathbf{M}, \mathbf{K} are calculated first by the Finite Element Method to form the system of equation as in (1). The flutter behavior is analyzed by the method described previously. In the Fig. 3a,b the frequencies and the damping measured by the logarithmic decrement of interested modes are plotted against the wind speed. It can be observed that the first mode becomes unstable at the wind speed of 59m/s. Further increasing the wind speed could lead to flutter on several modes with rather complicated behavior of mode switching. In general, the frequency of all modes show a tendency of decreasing with U .

This method of iteration, based on the vibration shape can be used to systematically study the variation of modes due to wind. The evolution of the first unstable mode is shown in the Fig. 4. It can be observed how the vibration shape starting from a torsional dominant mode with some swaying becomes vertical dominant with pitching. At very high wind, the vibration is practically heaving with insignificant torsion. It should be pointed out that the structure has at the same time a similar but highly damped mode. The only difference is the phase lag between the heaving and pitching motions

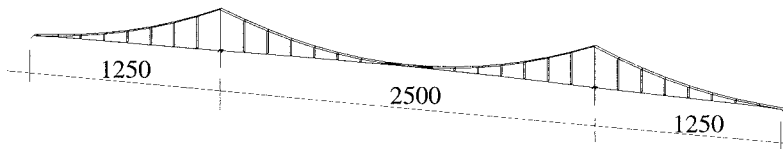


Fig. 2. Frame model of 2500m span length suspension bridge.

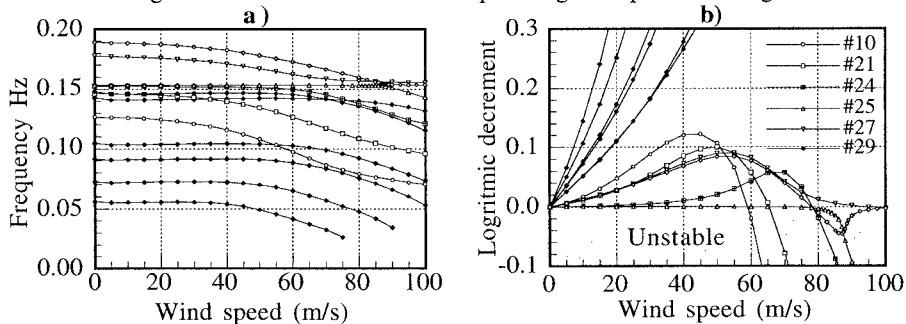


Fig. 3 The frequency a) and damping b) in function of the mean wind speed U .

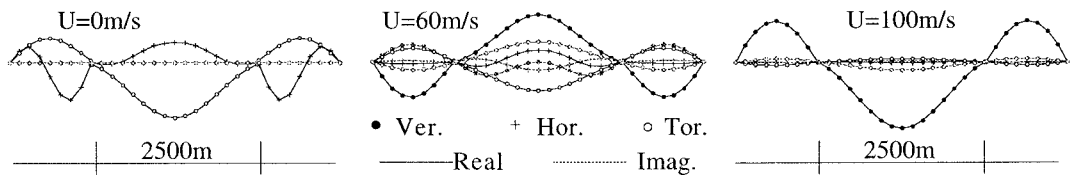


Fig. 4 The flutter mode shape of the mode #10 at different wind speed.

4. Concluding Remarks

The method of flutter analysis by mode tracing as presented has proved to be effective and computationally efficient. It is specially suitable for the bridge structure with very complex 3D behavior due to its dimension, the presence of cross, inclined hangers or with the active control built-in. The method also proved to be computationally efficient which could be easily applied to the flutter analysis of very large system.

References

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