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A circular rigid punch with one end sliding and the other end with a sharp corner on a cracked half plane acted by concentrated force

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1. **Introduction** A circular rigid punch with sharp corners in complete contact with a cracked half plane has been studied before[1], and the fundamental solution of the problem in [1] has also been derived recently[2]. The present paper deals with the problem of circular rigid punch with one end in smooth contact and the other end with a sharp corner on a cracked half plane acted by concentrated force at an arbitrary point on the boundary or in the body of the half plane. The length of the contact region is changed with the position of the concentrated force and other conditions, and is decided by the property that the stresses at the smooth end of the contact region must be finite. After the length of the contact region is decided, the stress intensity factors of the crack and the resultant moment on the contact region can be calculated with different positions of the concentrated force.

2. **The presentation of the problem** As shown in Fig.1, the punch is supposed to be vertical on a half plane with a vertical edge crack. To keep the punch not to incline, the position of the load on the punch is usually eccentric from the origin of the x-y coordinates to equilibrium the moment produced by the stresses on the contact region about the origin. The right end of the punch is with a certain distance of $a/2$ from the origin, while the left end of the punch is undetermined. Besides the load P on the punch, there also exists a pair of concentrated forces at point z_0 in the half plane. To solve the problem in an analytical way, the cracked half plane is mapped into a unit circle by the following rational mapping function[1]:

$$z = \omega(\zeta) = \frac{E_0}{1-\zeta} + \sum_{k=1}^{N} \frac{E_k}{\zeta_k - \zeta} + E_c \tag{1}$$

where E_0, E_k, ζ_k are known coefficients, and E_c is decided by the distance from the edge crack to the punch.

The loading and displacement conditions can be described as

$$p_x = p_y = 0 \quad \text{on} \quad L = L_1 + L_2 \tag{2a}$$

$$p_x = \mu p_y, \quad \int p_y ds = P \quad \text{on} \quad M \tag{2b}$$

$$V = x^2/2R \quad \text{on} \quad M \tag{2c}$$

$$Q(x,y) = (q_x + iq_y)\delta(z, z_0) \quad \text{on the surface or in the body of the half plane} \tag{2d}$$

where $L_1 = ABCD'D$, $L_2 = EA$, $M = DE$ in Fig.1, μ is the Coulomb's frictional coefficient on M; p_x and p_y represent the components of traction in x and y directions on the surface of the half plane; $Q(x,y)$ represents the concentrated forces on the boundary or in the body of the half plane; $\delta(z,z_0)=1$ when $z=z_0$ and 0 when $z \neq z_0$. V is the displacement of the punch, and R is the radius of curvature.

3. **The method of analysis** The complex stress functions of the problem are divided into two parts:

$$\phi(\zeta) = \phi_1(\zeta) + \phi_2(\zeta) \tag{3a}$$

$$\psi(\zeta) = \psi_1(\zeta) + \psi_2(\zeta) \tag{3b}$$

where $\phi_1(\zeta)$ and $\psi_1(\zeta)$ are the complex stress functions of the half plane with the edge crack acted by the concentrated forces[3]. $\phi_2(\zeta)$ and $\psi_2(\zeta)$ are the holomorphic parts of $\phi(\zeta)$ and $\psi(\zeta)$.

The general solution of $\phi_2(\zeta)$ can be expressed as [1,2]

$$\phi_2(\zeta) = H_1(\zeta) + H_2(\zeta) + H_3(\zeta) + \frac{1+i\mu}{2} J(\zeta) + Q(\zeta)\chi(\zeta) \tag{4}$$

where $H_1(\zeta)$, $H_2(\zeta)$, $Q(\zeta)\chi(\zeta)$ and $J(\zeta)$ are the same expressions as those in the previous paper[1], and $H_3(\zeta)$ is related to the concentrated forces in the half plane, which is expressed as

$$H_3(\zeta) = \frac{1-i\mu}{2}\frac{1}{2\pi}\left[(\bar{q} - \kappa q)F_1 + (\kappa\bar{q} - q)F_2 + qG_1 + \bar{q}G_2 + 2\pi G_3\right] \tag{5}$$

where F_1, F_2, G_1, G_2 and G_3 can be found in [2]. $q = -(q_x + iq_y)/(1+\kappa)$, $\kappa = 3-4\nu$ for plane strain and $(3-\nu)/(1+\nu)$ for plane stress state, and ν is the Poisson's ratio of the half plane.

4. **The length of the contact region** The stress components on the boundary of the half plane can be expressed as

$$\sigma_r + i\tau_{r\theta} = \frac{1}{\omega'(\sigma)}\left\{\frac{i(1-i\mu)P}{2\pi}\frac{(1-\alpha)(1-\beta)}{\chi(1)(1-\sigma)(\sigma-\alpha)(\sigma-\beta)} + \frac{e(\sigma)}{(\sigma-\alpha)(\sigma-\beta)} + \frac{mf(\sigma)}{\sigma-\alpha} + \frac{(1-m)f(\sigma)}{\sigma-\beta} + g(\sigma)\right\}\left[\chi^+(\sigma) - \chi^-(\sigma)\right] \tag{6}$$

where $e(\sigma)$, $f(\sigma)$ and $g(\sigma)$ are related to the concentrated forces in the half plane, which are known functions.

The following condition can then be formed from (6) to satisfy the finite stress condition at the smooth end:

$$\frac{i(1-i\mu)(1-\alpha)}{2\pi\chi(1)(\beta-\alpha)}P+\frac{e(\beta)}{\beta-\alpha}+(1-m)f(\beta)=0 \tag{7}$$

β can be obtained from (7), and then the length of the contact region can be decided. Fig.2 shows a' with different d, where d denotes the distance from the crack to the concentrated force on the surface of the half plane. It is shown that a' decreases and tends to a stable value with the increase of d.

5. The stress intensity factors of the crack The stress intensity factors of the crack are defined as

$$K_I-iK_{II}=2\sqrt{\pi}e^{-\frac{\delta}{2}i}\frac{\phi'(\zeta_0)}{\sqrt{\omega''(\zeta_0)}} \tag{8}$$

where $\delta=-\pi/2$ and $\zeta_0=-1$.

The non-dimensional stress intensity factors of the crack are defined as

$$F_I+iF_{II}=\frac{(K_I+iK_{II})}{P\sqrt{\pi}}\sqrt{a} \tag{9}$$

Fig.3 shows F_I and F_{II} with different d. F_I increases with the increase of d, and F_{II} is on the contrary. Both F_I and F_{II} tend to stable values with the increase of d.

6. The resultant moment The resultant moment about the origin of the x-y coordinates is calculated by

$$R_m=-\mathrm{Re}\left[\int_\alpha^\beta \omega(\sigma)\overline{\phi_i'}\left(\frac{1}{\sigma}\right)\frac{d\sigma}{\sigma^2}+\int_\alpha^\beta \overline{\omega}\left(\frac{1}{\sigma}\right)\phi_i'(\sigma)d\sigma\right] \tag{10}$$

The non-dimensional resultant moment is defined and the location of the load P is obtained as

$$M_r=\frac{R_m}{Pa}\ ,\qquad Pe=R_m \tag{11}$$

Fig.4 shows M_r with different d. M_r decreases with the increase of d from positive to negative value and tends to a stable value. The positive value of M_r corresponds to an anti-clockwise moment with the load P on the left side of y-axis.

In the above calculation, the concentrated forces are typically taken as $q_y/P=-1$ and $q_x=0$ acted on the surface of the half plane, $\kappa=2$, $\mu=0.5$, $b/a=0.5$, $c/a=0$ and $Ga^2/PR=1$ were selected.

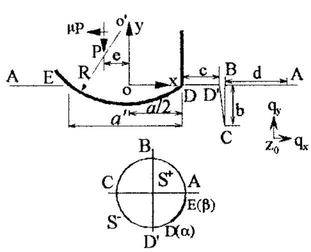

Fig.1 The punch and the unit circle

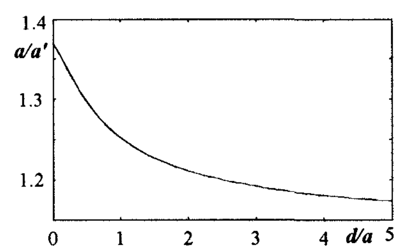

Fig.2 The length of the contact region

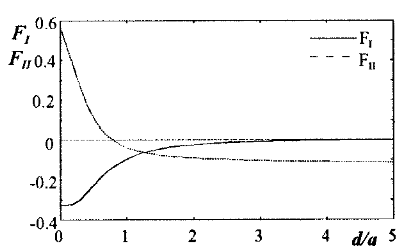

Fig.3 The stress intensity factors of the crack

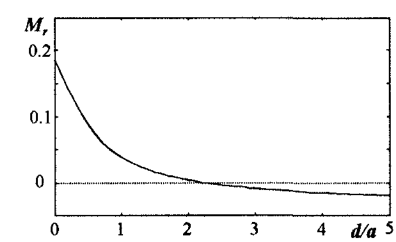

Fig.4 The resultant moment on the contact region

References

[1] Hasebe N. and Qian J., Contact Mechanics II, Southampton (1995)
[2] Qian J. and Hasebe N., 11th International Conference of BEM, Hawaii(1996)
[3] Hasebe N., Qian J. and Chen Y. Z., 11th International Conference of BEM, Hawaii(1996)