CS-53

Computation of Isothermal Flows with "Method C" Using Relaxation on Dirichlet Boundary Condition

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1 Introduction

Recently a lot of works were done using Finite Element Method, that was developed in Prof. Kawahara's laboratory and otherwise known as "Method C". This method is simple, fast, easy to use and understand and introducing various enhancements allows to overcome instability of this strait-forward Euler scheme. However the current formulation still has problems in dealing with pressure. Pressure Poison equation for this method uses Laplacian formulation and this requires at least one prescribed value on boundary. In many cases this value and it's location can be selected so it does not heavily affect the computation process. But in some cases this limitation does affect the computational process and introduces artificial numerical noise.

2 Pressure correction method

Pressure correction method for "Method C" was first proposed by Agnes Kovacs, Ref.[1]. The idea of this method is quite simple and effective. After initial pressure is obtained, prescribed value on the boundary is replaced by average, computed by certain formula using adjacent values of pressure from adjacent nodes. By relaxing Dirichlet boundary condition and updating prescribed values for pressure we avoid numerical dumping on pressure field. Even for very high velocities method works well. However when grid mesh in widely spaced the proposed approximation becomes inadequate and there is a need for improved approximation.

3 Basic equations

$$\frac{\partial U_i}{\partial t} + U_j U_{i,j} + \frac{1}{\rho} P_{,i} - Pr(U_{i,j} + U_{j,i})_{,j} = PrRaTF_i \tag{1}$$

$$\frac{\partial T}{\partial t} + U_j T_{,j} = T_{,jj} \tag{2}$$

With boundary conditions:

$$U_i = \hat{U}_i - onS_1 \tag{3}$$

$$(-\frac{1}{\rho} P_{,i}\delta_{ij} + Pr(U_{i,j} + U_{j,i}))N_j = \hat{t} - onS_2 \tag{4}$$

$$T = \hat{T} - onS_3 \tag{5}$$

$$T_{,j} N_j = \hat{\theta}_j - onS_4 \tag{6}$$

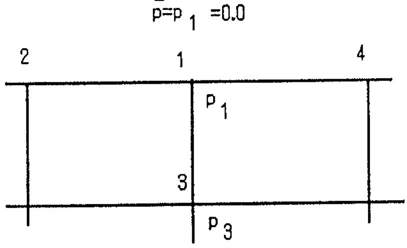
Where $Pr = \frac{\nu}{\alpha}$ is Prandtl number
$Gr = \frac{\beta g \Delta T L^3}{\nu^2}$ is Grashof number
$Ra = GrPr = \frac{\beta g \Delta T L^3}{\nu \alpha}$ is Rayleigh number

4 Relaxation of boundary condition

In short the idea of Kovacs method is relaxation of $\hat{p} = 0$ Dirichlet boundary condition. Value for p is not prescribed, but approximated using values of pressure from adjacent nodes. In four-node case we have

$$\hat{p^n} = p_1^n = (p_2^{n-1} + 0.5 * p_3^{n-1} + p_4^{n-1})/2.5 + p_t \quad (7)$$

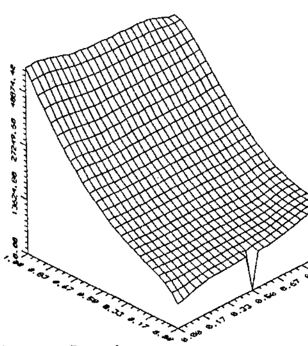
$$p_t = (p_2^{n-1} + p_4^{n-1} - p_2^{n-2} - p_4^{n-2})/2.0 \quad (8)$$



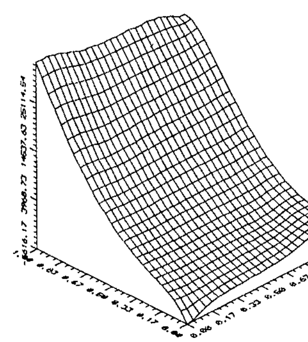
$\hat{p^n}$ denotes extrapolated values of $\hat{p}$ on n-th time step.

5 Numerical Study

To demonstrate advantages of present approach heat transfer in square cavity was chosen. $Ra = 10^5, Pr = 1.5$ Pic. 1 shows 3D pressure field without correction and Pic 2. with correction. As can easily be seen correction on pressure affects the entire field and makes it much more realistic and accurate.



Pic. 1



Pic. 2

6 Conclusions

We successfully applied relaxation of boundary condition to Isothermal Flows and demonstrated that this approximation works pretty well. This approach is recommended for use because of it's simplicity and effectiveness. However if the grid mesh is coarse and widely spaced current approximation becomes inadequate and needs improvement.

7 References

(1) A. Kovacs, M. Kawahara:"A finite element scheme based on the velocity correction method for the solution of the time-dependent incompressible Navier-Stokes equations", International Journal for Numerical Methods in Fluids, Vol. 13, pp403-423(1991)
(2) M. Hayashi, K. Hatanaka, M. Kawahara:"Lagrangian Finite Element Method for Free surface Navier-Stokes Flow Using Factional Step Methods", International Journal for Numerical Methods in Fluids, Vol. 12, pp805-840(1991)
(3) M. Marin, M. Kawahara: "Comparison between convetional Galerkin and Taylor-Galerkin methods for Isothermal Flows". Proc. Chuo University CFD symposium. Vol.8, pp667-670 (1994)