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Cracking Localization in Concrete Structures

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1. Introduction

Cracking localization is the process whereby small cracks, which are initially generated uniformly in concrete, can join to form a single large crack, which keeps growing and opening under increasing load. In reality, the localization of microcracking is examined and the beam fails due to one large crack, however such localization cannot be reproduced in conventional analysis. Whether many cracks are initiated at the initial stages or not is not yet clear. In fracture mechanics analysis, however, it is expected that cracking is first distributed in the beam, and is then localized into one dominant crack, which continues to grow and open with other smaller cracks being unloaded and closed. The present work assumes the existence of an unloading path in the tension-softening behavior of utmost importance to realize the phenomenon of cracking localization. The finite element method (FEM) for a body containing displacement discontinuity is considered as a suitable method to examine this phenomenon.

2. Model

The solid is assumed homogeneous and the behavior at any point is generally assumed to be isotropic linear elastic as long as the major principal stress is less than the tensile strength. The crack is formed when the maximum principal stress level at the element center reaches tensile strength in the direction normal to the maximum tensile stress. The tension-softening relation is employed upon the formation of crack with increasing crack opening displacement. Secant unloading is assumed for the crack closure phenomenon (see Fig. 1).

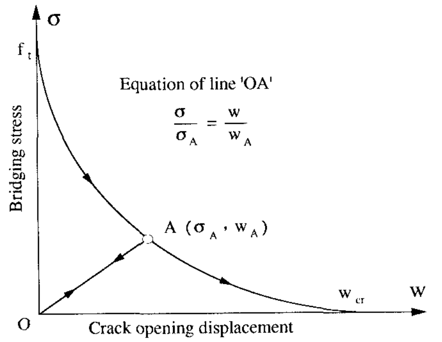

Fig. 1 Tension-softening relation

3. Method

Following formulation of tensile fracture phenomena [1], and piecewise polynomial basis functions [2] for a cracked element over two separate regions, the finite element equations for the 4-node cracked element (see Fig. 1), in general, can be written as

$$\begin{bmatrix} \int_\Omega \mathbf{B}^T\mathbf{D}\mathbf{B}d\Omega & \int_\Omega \mathbf{B}^T\mathbf{D}\mathbf{B}^C d\Omega \\ \int_\Omega \mathbf{B}^{C^T}\mathbf{D}\mathbf{B}d\Omega & \int_\Omega \mathbf{B}^{C^T}\mathbf{D}\mathbf{B}^C d\Omega + \int_{\Gamma^C} \mathbf{E}d\Gamma \end{bmatrix} \begin{Bmatrix} \Delta\mathbf{U} \\ \Delta\mathbf{U}^C \end{Bmatrix} = \begin{Bmatrix} \int_\Omega \mathbf{N}^T\Delta\mathbf{f}d\Omega + \int_{\Gamma_h} \mathbf{N}^T\Delta\mathbf{h}d\Gamma \\ \int_\Omega \mathbf{N}^{C^T}\Delta\mathbf{f}d\Omega + \int_{\Gamma_h} \mathbf{N}^{C^T}\Delta\mathbf{h}d\Gamma \end{Bmatrix}, \quad (1)$$

where $\Delta\mathbf{U}$ is the ordinary vector of incremental nodal displacements, $\Delta\mathbf{U}^C$ represents additional degrees of freedom corresponding to the crack, condensed at the element level, and are assumed to be constant within the element, $\mathbf{N}$ and $\mathbf{N}^C$ are the associated matrices of interpolation polynomials, $\mathbf{B}$ and $\mathbf{B}^C$ are the associated strain-displacement matrices, $\mathbf{D}$ is material stiffness matrix, $\Delta\mathbf{f}$ is the body force, $\Delta\mathbf{h}$ represents tractions on the cracked surface, and $\mathbf{E}$ is crack stiffness matrix. These finite element equations are implemented into a computer program [1]. In this method, at every step, a unit load is applied as the testing load, and for every uncracked element, the exact amount of load increment necessary for cracking the element, or, for change in slope of tension-softening curve for the cracked element is established in advance as explained elsewhere [1]. The minimum of the values is adopted as the load factor for the analysis. This process is continued until the complete piecewise linear load-displacement response is obtained.

4. Results and Discussion

To investigate the phenomena of cracking localization, an unreinforced concrete beam without a notch with dimensions as shown in Fig. 2 is considered. The present research assumes that the existence of an unloading path in the tension-softening behavior is of essential importance, and compares the two cases where the unloading path is either allowed for or neglected. In the first analysis, the unloading path is neglected. By incorporating this model in the program the analysis yields the load-displacement response as shown in Fig. 2, and the cracking pattern as depicted in Fig. 3, which clearly shows that the localization of microcracking cannot be reproduced.

In the second case, the unloading path is allowed for (see Fig. 1). Load-displacement response is depicted in Fig. 2. The crack pattern is shown in Fig. 4. In this case, all cracks, except one at the center, enter unloading paths and start to close, while the crack at the center continues to grow. The difference of these two cases shows the importance of the unloading path in the tension-softening behavior. Recently, Shirai [3] demonstrated the importance of cracking localization by comparing the results from various investigations, and showed that localization of microcracking exerts a significant influence on the response of beam specimens.

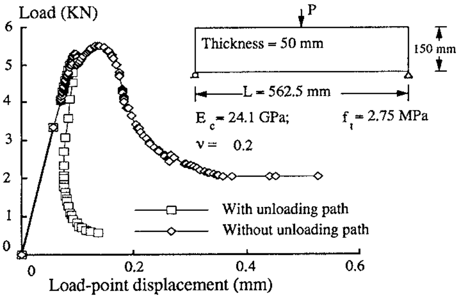

Fig. 2 Load-displacement response

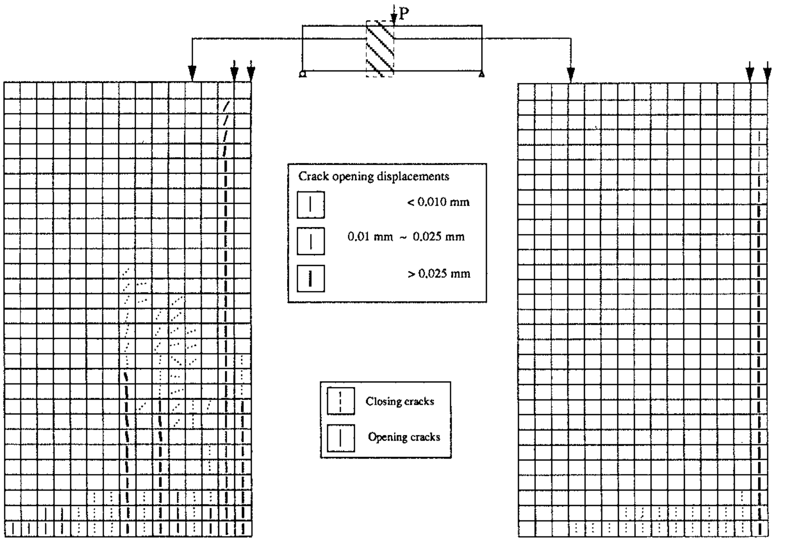

Fig. 3 Crack pattern without unloading path

Fig. 4 Crack pattern with unloading path

5. Conclusion

It is shown that unloading path in the tension-softening behavior of concrete plays a key role in the localization of microcracking in mode-I loading condition. Hence it may be concluded that the main reason for the inability of conventional analysis to describe cracking localization is either the use of material models not including unloading path or the lack of a suitable criterion for the initiation of unloading.

6. References

[1]. Ali, Asghar (1993)."FEM analysis of tensile fracture phenomena in concrete structures," Doctoral Dissertation, The University of Tokyo, Tokyo, Japan.

[2]. Dvorkin, E. N., Cuitino, A. M., and Gioia, G. (1990). "Finite elements with displacement interpolated localization lines insensitive to mesh size and distortions," Int. J. Num. Methods Engrg., 30, 541-564.

[3]. Shirai, N. (1994). "JCI round robin analysis of size effect in concrete structures," JCI International Workshop on Size Effect in Concrete Structures, H. Mihashi, H. Okamura, and Z. P. Bazant, eds., Sendai, Japan, E & FN Spon, 295-322.