

## Investigation on Optimality of A Proposed Optimal Shape Method

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## Introduction

With the interest on global shape change of structures, the authors proposed one optimal shape method previously [1], which offered the consideration on the characteristics of material. However, as it is known, the optimum point in a given domain may be either locally satisfied or globally unique [2]. It can be illustrated by the contour diagram shown in Fig.1(a) and Fig.1(b) in the case of two variables. In Fig.1(a), the global optimum point (P) can be located by starting from either A or B; P<sub>1</sub> and P<sub>2</sub> represent the local optimum points by searching from A and B respectively. Therefore, this study is motivated to investigate the uniqueness of the optimal solution in the proposed optimal method by the authors.

## Description of Investigation

For most of the structural optimization problems, the use of implicit function is generally introduced in the formulation of the optimization problem. Unlike the form of explicit function as shown in eq.1, implicit function is expressed in the form of eq.2.

$$y = f(\mathbf{x}) \quad (1) \quad ; \quad f(\mathbf{x}) = 0 \quad (2)$$

Generally speaking, it is hard to define the implicit function precisely. An alternative approach is often adopted by only dealing with the input and output variables, which is viewed as "the process of black-box". For the problems related to the interest of the proposed optimal method proposed by the authors, the control input variables can be supposed as follows: boundary conditions, load configuration, geometry of structure and material characteristics. However, due to use of numerical method (FEM), the related input variables also include: number of mesh, type of element and length of search vector. The procedure of this investigation is given in the following description. However, in order to get rid of the effect caused by the choice of different numerical methods (e.g. FEM or BEM), the control input variables related to the numerical methods are excluded. In addition, for simplifying the discussion, variations on variables are first made for only one variable at one time. If the uniqueness of the optimality of these cases is verified, the combination of variations on variables will be also discussed. For the discussion below, there are two kinds of material utilized, the mechanical properties of which are listed in Tbl.1.

## Examples of Investigation

The purpose of this investigation is to realize if the optimal solution obtained from the proposed method is unique or not. However, as we know, the variations on load configuration, boundary condition and material type will generate a new formulation of the optimization problem. (The details about these can be referred to [3]). Thus, the discussions for load configuration, boundary condition and material type are omitted. And all the examples given below are assumed with the same object function for minimizing the scatter of inner stress.

## (1) Frames with Different Ratio Of Column Width To Column Height

For understanding the effect of geometry of structure on the optimality of the proposed method, under the same load configuration (one-point concentration load), material type (concrete-like material with compressive strength( $F_c$ ): tensile strength( $F_t$ ) = 10 : 1) and boundary condition (the supports for both columns are hinged) as shown in Fig.2(a), the only difference in Fig.2(a) is the ratio of column width to column height (B/H). If the equivalent optimized shape could be obtained for both cases, the uniqueness of the optimality under these assumption can be accepted.

However, as compared in Fig.2(b), it is obvious to observe the difference existing between these two optimized results. In order to verify that these two structures have been optimized, the object functions for both structures are given in Fig.2(c), which indicates the convergence of the object functions. As for

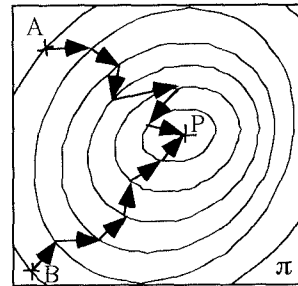


Fig.1(a) Global Optimum (P) in Domain  $\pi$

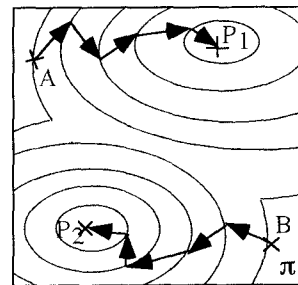


Fig.1(b) Local Optimum (P<sub>1</sub> and P<sub>2</sub>) in Domain  $\pi$

Tbl.1 Mechanical Properties of Material

	$F_c^*$	$F_t^*$	$E^*$	$\nu$
M1	400	400	150000	0.2
M2	400	40	150000	0.2

(\*unit : kgf/cm<sup>2</sup>)

the inconsistency of these two optimized structures, the reasons may resort to (a) precision of numerical approximation, which relates to the number of meshes and the type of element and (b) distortion of elements during the process of optimization. As shown in Fig.2(d), the maximum error varied with the process of optimization, which indirectly reveals the possible reason of the inconsistency.

## (2) Cantilever Beam with Different Cross-Section

The initial configurations of two structures are shown in Fig.3(a) and 4(a). The reason for choosing the wedge-like cantilever beam is its similarity to the optimized shape of a cantilever beam. The material used in this example is von Mises material with  $F_c : F_t = 1 : 1$ . Though the original shape of the wedge-like beam is close to the shape of optimized cantilever beam, its optimized result still shows some difference in the optimized cantilever beam as shown in Fig.3(b) and 4(b). It shows that the deviation value between approximate value based on numerical analysis and the true value indeed influences the reliability of the optimal solution even with good convergence of object function and acceptable error as shown in Fig.3(c), 3(d), 4(c) and 4(d).

## Conclusions

Based on the above investigation, the following conclusions can be extracted from this study : for the optimal solution obtained by the proposed method, only local optimum rather than global optimum can be obtained. The reasons for this deficiency of this optimal design method can result from the use of numerical approximation technique (FEM), the precision of which is critically influenced by the subdivision of structure, selection of element type. Due to intrinsic aspect of the proposed method, the distortion of element is generated along with the process of optimization, which give rises to the inaccurate result of analysis. As a result, in order to find out the global optimum of a optimal problem, high precision numerical technique with good performance on function of remesh is in need for being introduced in the proposed method.

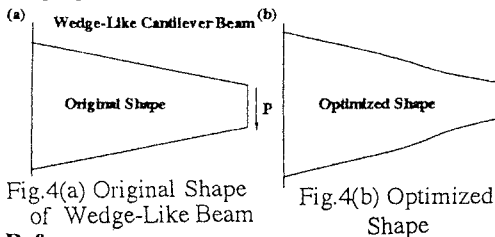


Fig.4(a) Original Shape of Wedge-Like Beam

Fig.4(b) Optimized Shape

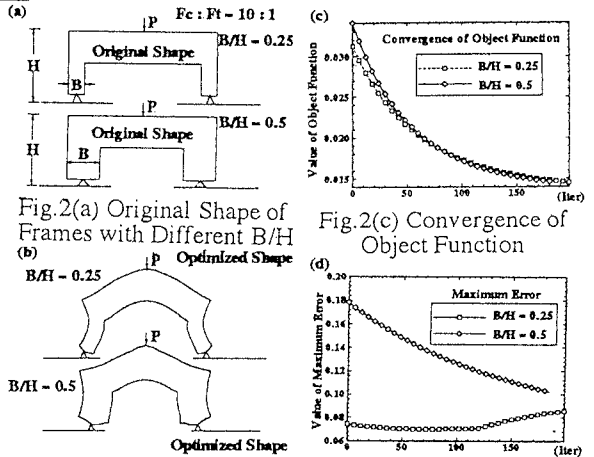


Fig.2(a) Original Shape of Frames with Different B/H

Fig.2(b) Optimized Shape of Frames with Different B/H

Fig.2(c) Convergence of Object Function

Fig.2(d) Maximum Error for the Frames

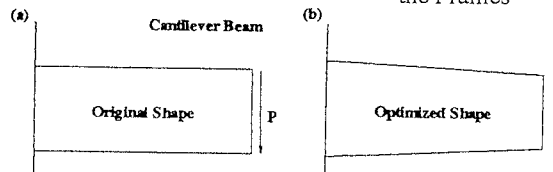


Fig.3(a) Original Shape of Cantilever Beam

Fig.3(b) Optimized Shape of Cantilever Beam

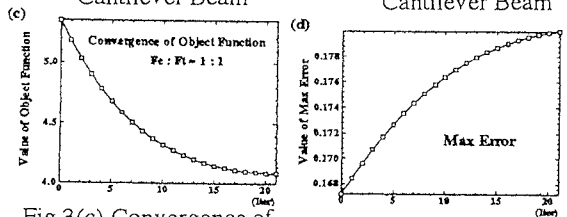


Fig.3(c) Convergence of Object Function

Fig.3(d) Maximum Error

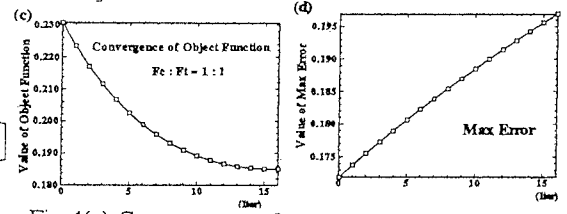


Fig.4(c) Convergence of Object Function

Fig.4(d) Maximum Error

## References

- [1] Hsu, K.L. and Uomoto, T., "A New Method for Optimal Shape Design of Structures", Proceedings of 49th Annual Conference of JSCE, Sec. 5, 1994, pp.832-833.
- [2] Reklaitis, G.V., Ravindran, A. and Ragsdell, K.M., *Engineering Optimization - Methods and Applications*, John Wiley & Sons, 1983, pp. 191-211.
- [3] Hsu, K.L., *A New Method for Optimal Shape Design of Structures Using Brittle Material*, MEng Thesis,, University of Tokyo, 1994.