

## Estimation of Model Parameters for Reconstructing Traffic Accidents

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### 1. Introduction

The accident reconstruction is classified into three phases; pre-impact, impact and post-impact. For driving simulation of pre-impact and post-impact phases, Sakai's tire model and two-wheel equivalence model were used [1,2]. There are four parameters in these driving models; friction coefficient, steering angle, slip ratio of front tires and slip ratio of rear tires. The impact model is based on the two dimensional impact model which was proposed by Ishikawa [3]. In this model, the normal and tangential restitution coefficients are defined at the impact center.

The main purpose of this work is to estimate the normal and tangential restitution coefficients in impact phase as well as the foregoing driving simulation parameters in post-impact phase. For this purpose, Box's complex algorithm method was applied[4]. To examine the validity of the Box's method, an artificial accident data was introduced. And then, the model parameters of actually observed traffic accident were estimated.

### 2. Theoretical models

#### 2.1. Impact model<sup>[3]</sup>

The schematic drawing of the two dimensional car-to-car impact model for analyzing accident reconstruction is shown in Fig. 1. In order to analyze the impact model, six equations are necessary: Four equations can be obtained from the law of conservation of linear and angular momentum. The last two equations are obtained from the constraint conditions at the impact center, in which normal and tangential restitution coefficients are defined.

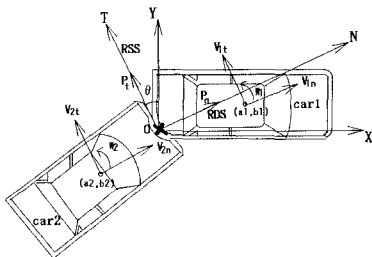


Fig. 1 Two dimensional impact model.

#### 2.2. Driving simulation models<sup>[1,2]</sup>

In this model the location of vehicle was calculated after each 0.01 second time interval. The Sakai's tire model was employed to obtain the friction forces between tire and surface of

road[2]. Then, two-wheel equivalence model was applied to calculate integrated forces of tires at the center of vehicles[3].

### 3. Model parameters and estimation method

We assume that the model parameters are known preliminary in the pre-impact phase. The parameters are unknown in both impact and post-impact phases. Based on the rest position of vehicles, we have to estimate the unknown parameters.

#### 3.1. Model parameters

1) **Impact model parameters.** We denote impact model parameters as follows:

$e_n$  = Normal restitution coefficient.

$e_t$  = Tangential restitution coefficient.

2) **Driving simulation parameters.** The parameters of driving simulation models are denoted as follows:

$u_i$  = Friction coefficient.  $H_i$  = Steering angle.

$S.F_i$  = Slip ratio of front tires.

$S.R_i$  = Slip ratio of rear tires.

$i$  = Number of vehicle.

#### 3.2. Box's complex algorithm<sup>[4]</sup>

In this method, we estimate the parameters so as to minimize the difference between calculated rest positions of vehicles and observed ones. The procedure of Box's method is a sequential search technique for finding a minimum objective function which is subjected to constraints as follows:

Minimize  $F_k(P)$

Subject to  $g \leq p \leq h$

where  $P = (e_n, e_t, u_1, H_1, \dots, S.F_2, S.R_2)$ .

$g, h$  = Constraint vector.

$k$  = Complex point.

The objective function is given below;

$$F_k(P) = \sum_{i=1}^2 \left( \frac{x_i - x_{si}}{x_{ri}} \right)^2 + \left( \frac{y_i - y_{si}}{y_{ri}} \right)^2 + \left( \frac{\theta_i - \theta_{si}}{\theta_{ri}} \right)^2 \quad (1)$$

where

$x_i, y_i, \theta_i$  = Calculated distance and yaw angle.

$x_s, y_s, \theta_s$  = Observed distance and yaw angle.

$x_r, y_r, \theta_r$  = Ranges of distance and yaw angle.

The unknown model parameters are subject to the following constraints;

$$-1 \leq e_n \leq 1, \quad -1 \leq e_t \leq 1$$

$$0 \leq u_i \leq 1, \quad -\pi/4 \leq H_i \leq \pi/4$$

$$0 \leq S.F_i \leq 1, \quad 0 \leq S.R_i \leq 1$$

### 3.3. Validity of Box's method

It was assumed that an accident, as shown in Fig. 2, occurred on a street. We specified the model parameters in advance. Fig. 2 shows the position of striking and struck cars based on the assumed model parameters. The initial, impact and rest positions are denoted by points 1, 2 and 3, respectively.

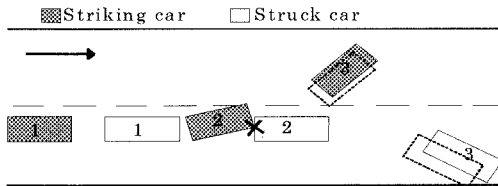


Fig. 2 Front to rear accident of two cars based on assumed data.

We estimated the model parameters in impact and post-impact phases. The model parameters were optimized by minimizing the difference between the calculated and the assumed rest positions. The dashed rectangles in Fig. 2 are the estimated rest positions by Box's method. The estimated parameters regarding this accident were compared with the assumed ones. As shown in Fig. 3, the difference between them was sufficiently small. It was found that the difference between calculated rest positions of vehicles and assumed ones was small for each vehicle.

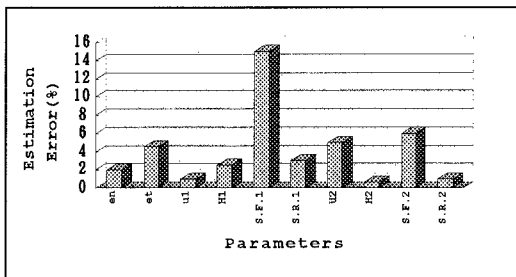


Fig. 3. The difference between assumed and estimated parameters as estimation errors.

### 4. Actual side impact accident

A side impact accident occurred at an intersection in Sapporo as shown in Fig. 4. The driving simulation of pre-impact phase was performed based on the data collected from the accident site. Then, we estimated the model parameters in impact and post-impact phases. Table 1 presents the estimated parameters by Box's method. The dashed rectangles in Fig. 4 are the rest positions based on the estimated parameters. The difference between the observed and the estimated rest positions is shown in Fig. 5 and 6. We can see that the difference in the positions of both vehicles is very small.

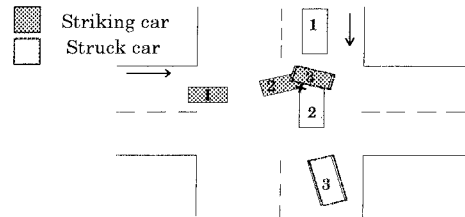


Fig. 4 Side impact accident based on observed data.

Table 1. Estimated parameters by Box's method.

$e_n$	$e_t$	$u_1$	$H_1$	$S.F_1$	$S.R_1$	$u_2$	$H_2$	$S.F_2$	$S.R_2$
-0.55	-0.64	0.48	2.86°	0.56	0.8	0.75	-31.5°	0.25	0.8

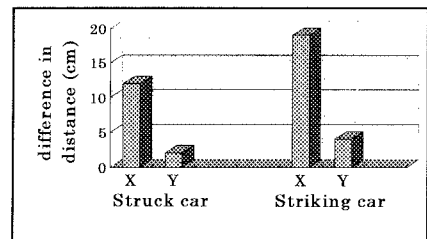


Fig. 5 Distance difference between observed rest positions and estimated ones.

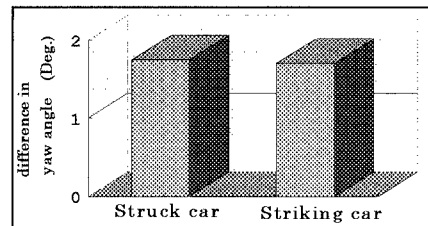


Fig. 6 Yaw angle difference between observed rest positions and estimated ones.

### 5. Conclusions

From this study the following conclusions can be drawn:

- 1) The impact model including restitution coefficients, combined with a driving simulation model, was applicable for reconstructing traffic accidents.
- 2) Box's complex algorithm was effective method in estimating of impact coefficients and driving simulation parameters for reconstructing traffic accidents.

### References

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