III - 134 Analysis of Pore Water Pressure by Various Constitutive Models

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1. Introduction

The generation and dissipation of pore water pressure is very important in soil mechanics, geotechnical engineering, and geoenvironmental engineering. A lot of works have been done on the problem, but there still is a big gap between the requirement and the research, this may be due to the fact that the different materials have different responses[2]. In this paper we will discuss the effect of different constitutive models on the dissipation of pore water pressure by means of Biot's consolidation theory[1,3]. From the discussion we can see that even almost the same response in derivatoric stress vs strain in drainage but a great difference exists in pore water pressure.

2. Components of Biot's theory incorporating with general constitutive models

Following six concepts form generalized Biot's theory[1,3]:

- Equilibrium equation

$$\frac{\partial \sigma_{ij}}{\partial x_j} + b_i = 0$$

- Geometrical equation

$$\Delta\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial \Delta u_i}{\partial x_j} + \frac{\partial \Delta u_j}{\partial x_i}\right)$$

- Constitutive laws of solid skeleton

$$d\sigma_{ij}' = D_{ijkl} d\varepsilon_{kl}$$

- Terzaghi's effective stress principle

$$\sigma_{ij} = \sigma_{ij}' + u\delta_{ij}$$

- Darcy's seepage law

$$q_i = K_{ij}\frac{\partial \psi}{\partial x_j}$$

- Incompressibility of solid-water mixture

$$\frac{\partial \varepsilon_v}{\partial t} = \frac{\partial q_i}{\partial x_i}$$

3. Weak form and FEM discretization

The body forces on the skeleton are two components: 1) The effective weight b_i per unit volume. 2) Seepage force induced by $(-u_{,i})$. So that the weak form for error-self-corrector mode is:

$$\int_V \{\delta(\Delta\varepsilon)\}^T[D]_{ep}\{\Delta\varepsilon\}dv - \int_V \{\delta(\frac{\partial \Delta \bar{u}_i}{\partial x_i})\}^T\{u\}^{t+\Delta t}dv - \int_V \{\delta(\Delta\bar{u})\}^T\{\Delta b\}dv$$
$$+ \int_{S_\sigma} \{\delta(\Delta\bar{u})\}^T\{n\}u^{t+\Delta t}ds - \int_{S_\sigma}\{\delta(\Delta\bar{u})\}^T\{\Delta\bar{T}\}ds$$
$$= -\int_V \{\delta(\Delta\varepsilon)\}^T\{\sigma\}dv + \int_{S_\sigma}\{\delta(\Delta\bar{u})\}^T\{\bar{T}\}ds - \int_V \{\delta(\Delta\bar{u}\}^T\{b\}dv \quad (1)$$

$$-\int_V \{\delta u\}^T\{\frac{\partial \Delta \bar{u}_i}{\partial x_i}\}dv = \frac{1}{r_w}\int_t^{t+\Delta t}[\int_{S_q}\{\delta u\}^T\{Ku\}ds]dt - \frac{1}{r_w}\int_t^{t+\Delta t}[\int_V\{\frac{\partial \delta u}{\partial x_i}\}^T\{K_i\frac{\partial u}{\partial x_i}\}dv]dt \quad (2)$$

For plane strain problems ($i = 1 \Rightarrow x$; $i = 2 \Rightarrow y$; element domain V_m), the displacement $\bar{u}$ and excess pore pressure u are taken to be the same shape function, 4-nodal isoparametric element. Crank-Nicolson mode ($\theta = \frac{1}{2}$) is adopted for time domain.

4. Constitutive Models of A Compacted Clay

4.1 Elastoplastic model

$$\{d\sigma\} = [D]_{ep}\{d\varepsilon\}, \qquad [D]_{ep} = [D] - \frac{[D]\{\frac{\partial g}{\partial \sigma}\}\{\frac{\partial f}{\partial \sigma}\}^T[D]}{\bar{A} + \{\frac{\partial f}{\partial \sigma}\}^T[D]\{\frac{\partial g}{\partial \sigma}\}} = [D] - \frac{G[X]}{\frac{\bar{A}}{G} + \Phi} \tag{3}$$

Here $[D]$ is elastic matrix; G: shear elastic modulus; $\bar{A}$: plastic hardening modulus; g: plastic potential function; f: yielding function; $\Phi = \{\frac{\partial f}{\partial \sigma}\}^T[D]\{\frac{\partial g}{\partial \sigma}\}/G$. Associated flow rule is used. The yield function f and hardening function H of a compacted clay are

$$f = (\frac{p' - H}{1.206H})^2 + (\frac{q}{1.46H})^2 - 1 = 0 \qquad H = \frac{P_a}{2.206}(\frac{\epsilon_v^p - 0.06\bar{\epsilon}^p}{0.0166})^{\frac{1}{0.5}} + \frac{P_r}{2.206} \tag{4}$$

$$\begin{cases} H \geq H_{max} & \text{:loading} \\ H < H_{max} & \text{:unloading/reloading} \end{cases} \tag{5}$$

P_r is the overconsolidation pressure, P_a is the atmospheric pressure. And elastic bulk modulus $B = 157.8p'$, and $G = 103.33P_a\left(\frac{\sigma_3}{P_a}\right)^{0.8514}$

4.2 Duncan-Change nonlinear elasticity[3]

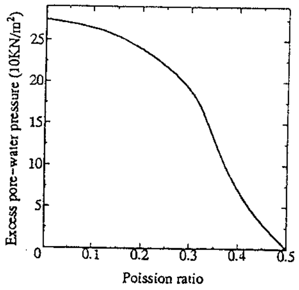
$$E_t = 20.2P_a\left(8 + \frac{\sigma_3}{P_a}\right)^{1.0944}\left[1 - \frac{0.7156(1 - \sin 30.7°)(\sigma_1 - \sigma_3)}{0.4P_a \cos 30.7° + 2\sigma_3 \sin 30.7°}\right]^2 \tag{6}$$

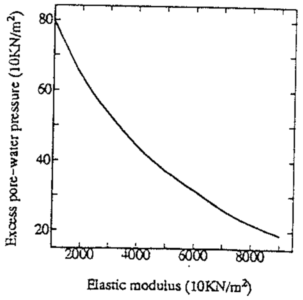
$$E_{ur} = 347.1P_a\left(\frac{\sigma_3}{P_a}\right)^{0.8304} \qquad B = 14.64P_a\left(8 + \frac{\sigma_3}{P_a}\right)^{1.08} \tag{7}$$

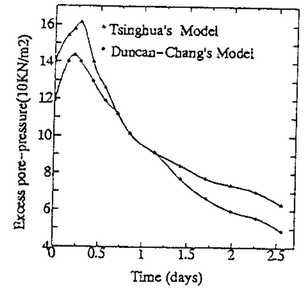
The permeability of this compacted clay is $K = 1.6 \times 10^{-7} m/s$.

5. Cases study

- One-dimensional consolidation
- Two-dimensional consolidation







6. Conclusions

- Dilatancy has a vital effect on excess pore water pressure. It is dangerous to ignore the positive dilatancy when the generation and dissipation of pore water pressure is considered.
- Loading level will affect the Mandel-Cryer effect.

References

[1] 市川康明(1990), 地盤力学における有限要素法入門, 日科技連出版社. [2] Jian-Guo Wang, T. Ito, and Yasuaki Ichikawa(1995) Effect of dilatancy on pore water pressure in porous medium, 第２６回岩盤力学に関するシンポジウム講演論文集(東京, 1995 年１月 19 日 — 20 日), pp.480-484 [3] Wang, J.-G.(1994), Instructions on COND2.FORT(2-DIM), Ichikawa's Lab., Dept. of Geotechnical and Environmental Engineering, Nagoya University