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A One-dimensional Irregular Wave Calculation Based on the Boussinesq Equations

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Deformations of regular waves over constant water depth as well as slope have been investigated in some papers based on Boussinesq equations. However the simulation for irregular wave is still limited. In this paper deformations of a two-component wave and one-dimensional irregular wave are calculated based on the Boussinesq equations. The model is verified in terms of dispersive property by comparing the results of simulation of a two-component wave propagating over a uniform slope with the laboratory data of Okayasu and Matsumoto (1995). Computation of irregular waves propagating over a uniform slope is also carried out and the results are compared with above experiment data. Good agreement is obtained.

I. Introduction of the wave model

The governing equations are the two-dimensional Boussinesq type equations and the continuity equation (Kabiling and Sato,1994).

$$\frac{\partial \eta}{\partial t}+\frac{\partial Q_x}{\partial x}+\frac{\partial Q_x}{\partial y}=0 \tag{1}$$

$$\frac{\partial Q_x}{\partial t}+\frac{\partial}{\partial x}\left(\frac{Q_x^2}{d}\right)+\frac{\partial}{\partial y}\left(\frac{Q_x Q_y}{d}\right)+gd\frac{\partial \eta}{\partial x}=\frac{1}{3}h^2\left(\frac{\partial^3 Q_x}{\partial x^2 \partial t}+\frac{\partial^3 Q_y}{\partial x \partial y \partial t}\right)-\frac{f_w}{2d^2}Q_x\sqrt{Q_x^2+Q_y^2} \tag{2}$$

$$\frac{\partial Q_y}{\partial t}+\frac{\partial}{\partial x}\left(\frac{Q_x Q_y}{d}\right)+\frac{\partial}{\partial y}\left(\frac{Q_y^2}{d}\right)+gd\frac{\partial \eta}{\partial y}=\frac{1}{3}h^2\left(\frac{\partial^3 Q_x Q_y}{\partial x\, \partial y\, \partial t}+\frac{\partial^3 Q_y}{\partial y^2\, \partial t}\right)-\frac{f_w}{2d^2}\sqrt{Q_x^2+Q_y^2} \tag{3}$$

where Q_x and Q_y are the average flow rate over the water depth in x and y directions respectively, η is the water surface elevation, and f_w is the bottom friction factor.

The wave breaking index of Isobe,M. (1986) is used to determine the location of wave breaking. It is expressed as

$$\gamma_b' = \left(\frac{u}{c}\right)_b = 0.53-0.3\exp(-3\sqrt{d_b/L_0}) + 5\tan^{3/2}\beta\exp\left\{-45(\sqrt{d_b/L_0}-0.1)^2\right\} \tag{4}$$

On the basis of the fact that random waves break earlier than regular waves, the index u/c is discounted by a factor of 0.8. According to long wave theory, the breaking index of $(u/c)_b$ is approximately equal to $(\eta/d)_b$. In the computation, spatial distribution of water surface elevation is obtained at every time step. Once waves are judged to be breaking, energy dissipation terms proposed by Sato et al. (1992) are introduced. They are

$$M_{Dx} = -\nu_e\left(\frac{\partial^2 Q_x}{\partial x^2}+\frac{\partial^2 Q_x}{\partial y^2}\right) \tag{5}$$

$$M_{Dy} = -\nu_e\left(\frac{\partial^2 Q_y}{\partial x^2}+\frac{\partial^2 Q_y}{\partial y^2}\right) \tag{6}$$

where ν_e is computed from the following formula,

$$\nu_B = \frac{\alpha_D\, gd\tan\beta}{\sigma^2}\sqrt{\left(\frac{g}{d}\right)\left(\frac{\hat{Q}-Q_r}{Q_s-Q_r}\right)} \tag{7}$$

where α_D is a coefficient which has the maximum value 2.5 at the crest of breaking wave and 0 at the crest of non-breaking wave as done by Kubo et al.(1992). The waves are determined by zero-downcrossing method.

The governing equations are solved by a space-staggered ADI method. Time series of water surface elevation of irregular wave is input at the offshore boundary. The reflected waves coming from the computational domain are estimated from the nearest point inside the domain by the simple relation of $\eta = \eta_{in} + \eta_{out}$ and then the reflected wave η_{out} is superposed to the incident water surface elevation. The down end boundary is set to allow waves passing through freely. The two side boundaries are set to be fully reflective.

II. Verification of the model

In order to examine the dispersive property of the wave model, propagation of wave series over a uniform slope of 1/30 is computed. The input wave series is the measured water surface elevation at offshore which is generated from two harmonics with the same wave height of 0.048m and different wave periods of 1.0s and 1.2s. The

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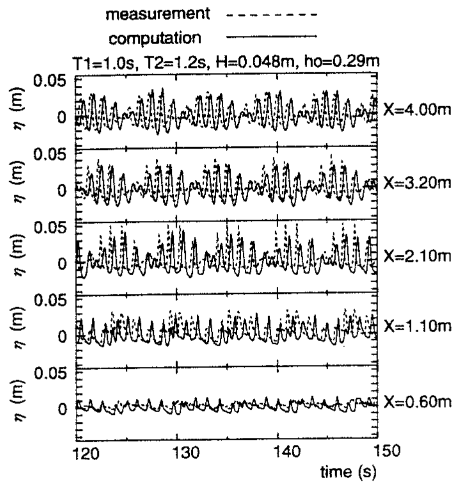
offshore water depth is 0.29m. The computational results and comparison with the experiment data of Okayasu and Matsumoto (1995) are shown in Figure 1. It can be said that there is a good agreement between the calculation and the measurement.



X : distance from shoreline
η : water surface elevation
Figure 1: Water surface elevation of a two-component wave propagating over constant slope.

Irregular waves propagating over a uniform slope of 1/30 are also calculated. The significant wave height and wave period are 3.91cm and 1.01s respectively and the constant water depth at the incident side is 29cm. The input wave series is a superposition of 20 harmonics, the frequencies of which are determined from Bretchneider-Mitsuyasu type spectrum and the largest frequency is 1.88. Comparison of the significant wave height between computation and measurement is shown in Figure 2. Good agreement is obtained.

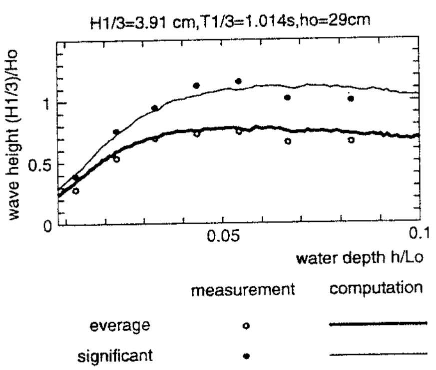


Figure 2: Comparison of significant wave heights.

The spatial distributions of water surface are shown in Figure 3, in which we can see that the wave propagation is properly simulated.

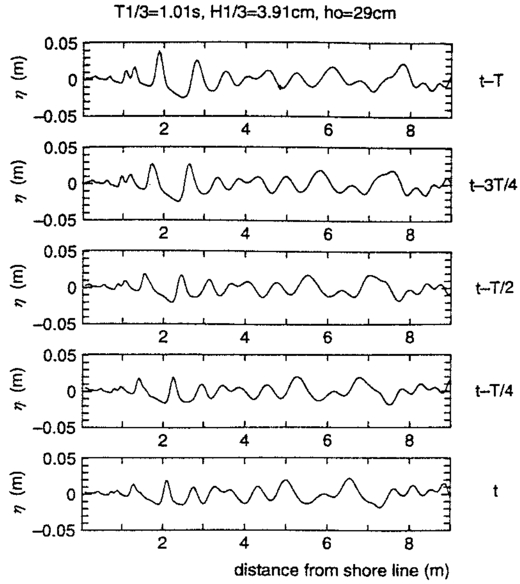


Figure 3: Water surface of irregular wave deformation over uniform slope.

III. Conclusion

The above results show that Boussinesq equations are possible to be used in the simulation of irregular wave deformation and the present model can simulate irregular waves propagating over constant slope and constant water depth properly.

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