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Dynamic Green's Functions For Semi-infinite Poroelastic Medium

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Introduction

The development of an efficient method to calculate dynamic Green's function for poroelastic medium can have significant implication in fields such as earthquake engineering and dynamic soil mechanics. The Green's functions may be used to determine the response of arbitrarily shaped foundations when excited by external forces as well as incoming seismic waves.

The main object of this paper is to presented a set of 3-dimensional Green's functions corresponding to a time-harmonic concentrated loading applied on the traction free surface of a dissipative semi-infinite poroelastic medium. Biot's equations for dynamic poroelasticity with internal friction between solid phase and fluid phase are considered.

Mathematical Formulation

1. General Governing Equations: Using axisymmetric displacement-potential form, the displacements of porous medium in terms of solid phase displacement and relative displacement of fluid phase in cylindrical coordinates are written as:

$$\{u,w\}=\{(u_r,u_z),(w_r,w_z)\}=\{(\nabla\phi+\nabla*\nabla*(\psi i_3)),(\nabla H+\nabla*\nabla*(Gi_3))\} \tag{1}$$

where, u_r,u_z=radial, vertical displacement of solid phase respectively. w_r,wz=radial, vertical displacement of fluid phase relative to solid ones, respectively, ∇=gradient and i_3=a unit base vector along z axis.

In the above equations, ϕ,H are the solid phase displacement potentials and ψ,G are potentials of the relative displacement between solid and fluid phases. Variations of ϕ,ψ are associated with volumetric wave propagation, whereas H,G show distortional wave propagation. The Eq.(1) can be rewritten to the motion governing equations as bellow:

$$\begin{bmatrix}\lambda^*+2\mu^*+Q & Q\\ Q & Q\end{bmatrix}\nabla^2\begin{Bmatrix}\phi\\ \psi\end{Bmatrix}=\begin{bmatrix}\rho & \rho_f\\ \rho_f & \alpha\rho_f/n\end{bmatrix}\frac{\partial^2}{\partial^2 t}\begin{Bmatrix}\phi\\ \psi\end{Bmatrix}+\begin{bmatrix}0 & 0\\ 0 & b\end{bmatrix}\frac{\partial}{\partial tt}\begin{Bmatrix}\phi\\ \psi\end{Bmatrix} \tag{2}$$

$$\begin{bmatrix}\mu^* & 0\\ 0 & 0\end{bmatrix}\nabla^2\begin{Bmatrix}H\\ G\end{Bmatrix}=\begin{bmatrix}\rho & \rho_f\\ \rho_f & \alpha\rho_f/n\end{bmatrix}\frac{\partial^2}{\partial^2 t}\begin{Bmatrix}H\\ G\end{Bmatrix}+\begin{bmatrix}0 & 0\\ 0 & b\end{bmatrix}\frac{\partial}{\partial tt}\begin{Bmatrix}H\\ G\end{Bmatrix} \tag{3}$$

where, $\lambda^*=\lambda(1+id)$, $\mu^*=\mu(1+id)$, λ and μ=Lame's constants, d=hysteresis damping ratio of solid phase, $\rho=(1-n)\rho_s+n\rho_f$ =density of soil, ρ_s=density of solid, ρ_f=density of fluid, n=porosity, $Q=1/(n(1/k_f+(1-s)/p))$, p=atmospheric pressure, k_f=bulk modules of fluid, s=degree of saturation, $\alpha=2/n-1$=toruosity for spherical shapes, showing mass coupling effect of phases, $b=gn\rho_f/k$=diffusive coefficient, g=gravitational acceleration, k=permeability coefficient of soil.

In steady state, after some mathematical manipulation[1], the stiffness coupling Eq.(2) and the mass coupling Eq.(3) can be uncoupled into two independent wave equations respectively. By using Hankel transformation and considering radiation condition, the solutions of these reduced wave equations can be obtained as:

$$\{\eta_1',\eta_2',\eta_3'\}=\{A\exp(-p_1z),B\exp(-p_2z),C\exp(-p_3z)\} \tag{4}$$

where, η'_i=zero-th order of Hankel transformation of η_i, $p_j=\sqrt{k^2-\beta_j^2}$, $j=1,2,3$ and k=wave number, respectively. The unknown coefficients A, B, C, must be determined to satisfy the boundary conditions.

2. Green's Functions: The displacement responses of a fluid-saturated poroelastic halfspace to a vertical harmonic unit loading applied on the surface are shown in Fig.-1. This surface is assumed in traction-free and drained condition. Therefore the displacement of the soil is composed of the displacement of

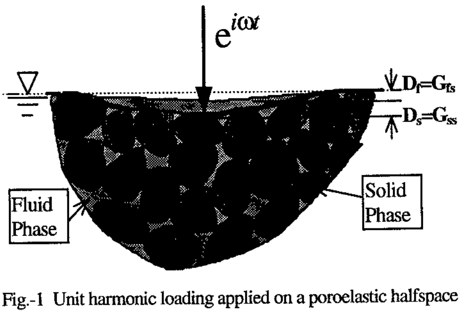

Fig.-1 Unit harmonic loading applied on a poroelastic halfspace solid phase (D_s) and the displacement of fluid phase (D_f). Actually these two displacements can be taken as the Green's functions of solid phase (G_{ss}) and fluid phase (G_{fs}) respectively.

The unit point loading function can be expressed as a combination of a δ function and the distributed loading along the circle on the surface[2].

$$F(r)=\frac{\delta(r)}{2\pi r} \quad (5)$$

When the radius of circle r_0 tends to zero, the limitation of $F(r)$ becomes the magnitude of the unit harmonic loading, as shown in Fig.-2.

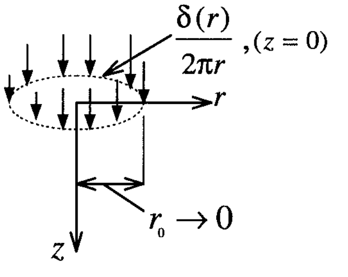

Fig.-2 Unit Point loading model

Based on the aforementioned assumptions, the stress condition on the surface can be taken as follows:

$$\sigma_{zz}=\frac{\delta(r)}{2\pi r},\ \tau_{rz}=0,\ p=0,\ (z=0) \quad (6)$$

where, σ_{zz}=normal stress in z direction, τ_{rz}=shear stress in r direction, p=fluid pressure, respectively.

From Eq.-(4) the stresses and displacements in wave-number domain can be given as below:

$$H_0(\sigma_{zz})=(\lambda^*+2\mu^*)(Ap_1^2\exp(-p_1z)+Bp_2^2\exp(-p_2z)-k^2p_3C\exp(-p_3z))-k^2\lambda^*(A\exp(-p_1z)+B\exp(-p_2z)-Cp_3\exp(-p_3z))$$

$$H_1(\tau_{rz})=\mu^*k(2Ap_1\exp(-p_1z)+2Bp_2\exp(-p_2z)-C(p_3^2+k^2)\exp(-p_3z))$$

$$H_0(P)=Q_f((1+t_{21})\beta_1^2A\exp(-p_1z)+(1+t_{22})\beta_2^2B\exp(-p_2z)) \quad (7-a)\sim(7-c)$$

$$H_0(u_z)=-p_1A\exp(-p_1z)-p_2B\exp(-p_2z)+k^2C\exp(-p_3z) \quad (8-a)$$

$$H_0(w_z)=-p_1t_{21}A\exp(-p_1z)-p_2t_{22}B\exp(-p_2z)+(\alpha_1/\alpha_2)k^2C\exp(-p_3z) \quad (8-b)$$

By substituting the Eq.-(6) into Eq.-(7-a)~(7-c), the unknown coefficients A, B, C above are determined as:

$$A=H_0(f_j)(2k^2-\beta_3^2)/\mu^*R_3,\quad B=-R_1A,\quad C=R_2A \quad (9-a)\sim(9-c)$$

where, R_1,R_2=intermedia variate, R_3=Rayleigh function for the porous medium.

Finally by substituting Eq.-(9-a)~(9-c) to Eq.(8-a)~(8-b) and using inverse Hankel transformation, we obtain the Green's functions as.

$$G_{ss}=u_z=(1/\mu^*)\int_0^\infty\frac{1}{2\pi R_3}\{-p_1(2k^2-\beta_3^2)+p_2R_1(2k^2-\beta_3^2)+2k^2(p_1-R_1p_2)\}kJ_0(kr)dk \quad (10)$$

$$G_{fs}=w_z=(1/\mu^*R_3)\int_0^\infty\frac{1}{2\pi}\{-p_1t_{21}(2k^2-\beta_3^2)+p_2t_{22}R_1(2k^2-\beta_3^2)+2k^2\frac{\alpha_1}{\alpha_5}(p_1-R_1p_2)\}kJ_0(kr)dk \quad (11)$$

Conclusion

Based on Biot's theory the dynamic Green's functions for a semi-infinite poroelastic medium are obtained. By using numerical method, the integrals of the Green's functions, Eq.-(10) and Eq.-(11) are evaluated. To conduct an efficient and accurate numerical calculation, trapezoidal rule with adaptive choice of stepwise is used. The numerical results of the Green's functions will be shown at the coming presentation.

Reference

[1] Nourzad,A.,"Dynamic interaction between a rigid body and the surrounding semi-infinite poroelastic medium", doctoral thesis presented to the University of Tokyo, 1994. [2] Graff Karl F., "Wave Motion in Elastic Solids", Dover Publication, 1991.