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# Nonlinear Mode Localization of Two-Degree-of-Freedom Systems

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## 1. Introduction

The mode localization phenomena in linear systems have been recently investigated. It was shown in [1] that weak structural irregularities may lead to a confinement of the free and forced vibrations in weakly coupled linear periodic systems. That is, the vibration modes of those systems possess a limited number of substructures being largely excited. In this study, the symmetric two-degree-of-freedom systems with closely-spaced natural frequencies and weak nonlinearity in supporting spring stiffness are studied. It will be shown that the nonlinear systems can exhibit mode localization phenomena in which one of the oscillators vibrates with large amplitude while the other is nearly at rest.

## 2. The Two-Degree-of Freedom Systems

The vibration of the two-degree-of-freedom system as shown in Fig. 1 is studied. The system consists of two identical single-degree-of-freedom suboscillators, with a unit mass $m$, connected by means of a coupling stiffness, $k_c$. The coupling stiffness is assumed to be small so that the system can possess closely-spaced natural frequencies. Each suboscillator is connected to rigid support by a massless spring having a linear stiffness, $k_s$, and a nonlinear hardening type stiffness of cubic order, $\hat{\alpha}$. $\omega=\sqrt{k_s/m}$ is the linear natural frequency of an oscillator. The undamped system is studied for the steady state responses of free vibration case while the damped system is used to investigate the bounded steady state responses of harmonically forced vibration. The equations of motion are

$$\ddot{x}_1+\omega^2 x_1+\hat{\alpha}x_1^3+k_c(x_1-x_2)+\hat{\mu}\dot{x}_1=F\cos(\Omega t) \qquad (1a)$$

$$\ddot{x}_2+\omega^2 x_2+\hat{\alpha}x_2^3+k_c(x_2-x_1)+\hat{\mu}\dot{x}_2=F\cos(\Omega t). \qquad (1b)$$

Note that, for the case of free vibration, the excitation terms and damping terms will vanish.

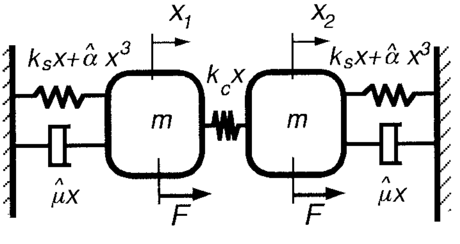


Fig. 1. Two-degree-of freedom systems

## 3. Perturbation Technique

The solutions of the nonlinear coupling equations are approximately evaluated by perturbation method. The responses of system are assumed to be in the form of

$$x_i(t)=x_{i0}(t)+\varepsilon x_{i1}(t)+O(\varepsilon^2), \qquad (2)$$

where $i$=1, 2 and $\varepsilon$ is a small dimensionless parameter. For a weakly nonlinear spring, a weakly coupling stiffness and lightly damped suboscillator, one puts $\hat{\alpha}=\varepsilon\alpha$, $k_c=\varepsilon k$ and $\hat{\mu}=\varepsilon\mu$. Moreover, to obtain a uniformly valid approximate solution of the problem, the excitation should be in the order that it will appear when the damping and the nonlinearity appear during perturbation. To accomplish this, one scales $F=\varepsilon f$. The interest of harmonically forced response lies in the neighborhood of primary resonance, i.e. $\Omega\approx\omega$. Thus, one lets $\Omega=\omega+\varepsilon\sigma$, where $\sigma$ is a detuning variable. The method of multiple scales [2] can be applied to the problems in order to obtain a uniformly valid, first order approximation to the dynamic response of the system. The responses of both vibration cases are in the form

$$x_i=a_i\cos\left(\omega t+\beta_i\right)+\varepsilon\frac{\alpha}{32\omega^2}a_i^3\cos\left(3(\omega t+\beta_i)\right)+O(\varepsilon^2) \quad , i=1,2, \qquad (3)$$

where $a_i$ and $\beta_i$ are the amplitude and phase of the response, respectively. The characteristics of steady state responses will be discussed in the next section.

## 4. Numerical Examples

For weak nonlinearity and weak coupling with the finite value of the ratio of coupling to nonlinearity, the nonlinear support stiffness and coupling stiffness terms for forced vibration case are selected as $\hat{\alpha}=0.01\,k_s$ and $k_c=0.01\,k_s$ while these parameters are arbitrary, but small, for free vibration case. The others parameters are $\varepsilon=0.001$, $\hat{\mu}=0.02$ (1% damping ratio), $\omega=1.0$ and $F=0.1$ .

The vibration confinement can be investigated from the amplitudes of steady state response, $a_i$ . It is well known that the normal modes of oscillation of a periodic multi-degree-of-freedom system, consisted of identical subsystems, is extended throughout the system. The amplitude of each suboscillator varies sinusoidally with its position in space. As a result, the amplitudes of both oscillators in a periodic two-degree-of-freedom system considered here are equal for the extended modes. In addition, the mode localization phenomenon can be observed from this system when there is a difference between the amplitudes of two oscillators. The results of example problems are shown in Fig. 2 for free vibration and Fig. 3 for harmonically forced vibration. In Fig. 2, the abscissa is the ratio of coupling stiffness to nonlinearity while the ordinate is the ratio of amplitude of the two oscillators. There exist two types of solution. Firstly, the solution of the extended mode in which two masses vibrate at the same amplitude is shown as the line $a_2/a_1=1$ . This solution is common with the linear system. The other interesting solution is the mode localization solution in which each mass vibrates with different amplitudes. The phenomena become more evident, ratio of amplitudes tends to zero or infinity, when the coupling stiffness decreases and the degree of nonlinearity increases. The results shown in Fig. 2 are all orbitally stable, so they can be solutions of oscillation depending on the initial conditions of the system.

The responses of harmonically forced vibration is shown in Fig. 3, together with response of linear system. The abscissa is the excitation frequency written in term of detuning parameter, $\sigma$. The frequency response curve shows the jump phenomena, which is the important characteristic of nonlinear system. In the normal extended mode, both oscillators vibrate with the same amplitude so that they can be shown in the same line in frequency response curve in Fig. 3. In addition, there is another mode of vibration having different steady state response amplitude of two oscillators. This mode is the localization phenomena occurred at the possible region of jump phenomena. It is seen that one of the oscillator vibrates with large amplitude close to the upper curve while the other oscillator vibrates with much smaller amplitude close to lower curve of the frequency response. It should be noted that the mode localization are orbitally stable and the larger amplitude can be either $a_1$ or $a_2$, depending on the initial conditions of system. The mode localization phenomena investigated in the linear perturbed systems [1] are also observed in this work in the nonlinear periodic system. The localization occurs when the nonlinear uncoupled system reduces to a set of weakly perturbed oscillator, due to the fact that the frequencies of oscillation of nonlinear systems generally depend on their amplitudes of vibration.

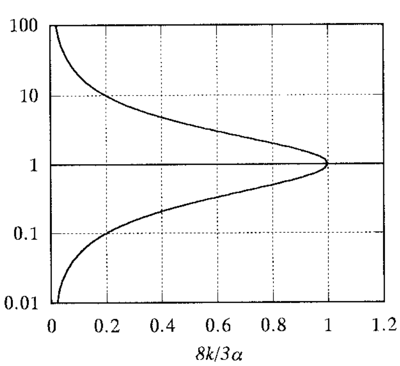


Fig. 2. Steady state amplitudes ratio of free vibration

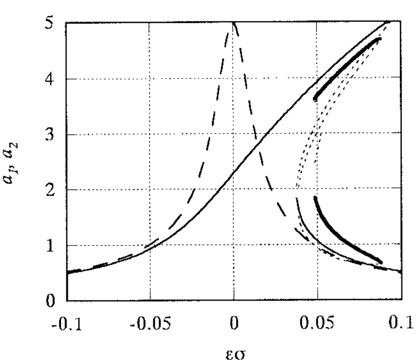


Fig. 3. Frequency response curve; (— —) linear system, (———) extended mode, (———) localization mode, (– . – .) unstable solutions

## 5. Concluding Remarks

This work introduces the important phenomena of vibration localization caused by nonlinearity. It was shown that the vibration amplitudes of symmetric two-degree-of-freedom system with structural nonlinearity are different in some cases. One of the oscillator vibrates with much large amplitude compared with the other. This localization of vibration may lead to failure of structure. On the other hand, the phenomena is favorable from view point of vibration isolation.

## References

1) Hodges, C. H. : Confinement of vibration by structural irregularity, Journal of Sound and Vibration, Vol. 82, No. 3, pp. 411-424, 1989

2) Nayfeh, A. H. and Mook, D. T.: Nonlinear Oscillations, John Wiley & Sons, 1979