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A MODEL TO STUDY PROJECTILE PENETRATION IN SOILS

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ABSTRACT: A mathematical model for one dimensional analysis for projectile penetration into earth media have been proposed. This model is based on "cylindrical cavity expansion" concept which reduces the problem in one dimensional. Using the principle of conservation of mass and momentum, basic equations, related to displacements (strains) and forces (stresses) have been developed. These are solved with Runge-Kutta self iterative method. The study is useful in design of nuclear power plants and other strategic structures at a safe depth below earth against any possible missile or terrorist attack.

Modeling: Being a complex problem certain assumptions are always enviable, only axisymmetric penetration is analyzed. Heat transfer, dissipation and friction is neglected. Projectile is considered a rigid body with slender nose and only plastic strains are taken in account.

As shown in fig. (2) the axis of symmetry (vertical axis) is divided into finite differences and the soil medium is divided into discs by planes being normal to the axis of symmetry. Each soil disc has infinite dimension in the radial direction and a constant thickness Δz which is equal to the finite difference measure or step along the axis of symmetry. Vertical component of displacement of each soil particle is negligible and only radial component is considered.

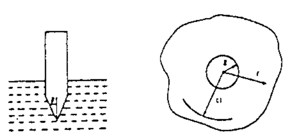

Fig. 1 Geometry of the problem

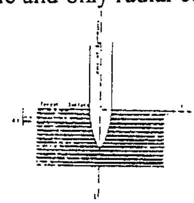

Fig. 2 Continuum idealisation by soil discs

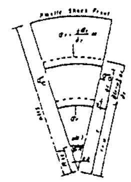

Fig. 3 A sector of a typical disc.

Formulation: Governing equations are derived in cylindrical, Lagrangian coordinates. With the aid of fig. (3) the equations of momentum and mass conservation are

$$\rho_0 r\frac{\partial^2 u}{\partial t^2} = -(r+u)\frac{\partial \sigma_r}{\partial r} - (\sigma_r - \sigma_c)\frac{\partial}{\partial r}(r+u) \qquad \text{...(1a)}$$

$$\rho_0 r = \rho(r+u)(1+\frac{\partial u}{\partial r}) \qquad \text{...(1b)}$$

where ρ_0 and ρ are the initial and current densities, u is the radial displacement , σ_r and σ_c are the radial and circumferential component of Cauchy stress (taken positive in compression) respectively for the target. Material is described by the hydrostat

$$p = K(1-\rho_0/\rho) = K\eta \qquad \text{...(2a)}$$

and the shear failure-pressure relation

$$\tau = \sigma_r - \sigma_c = \mu p \qquad \text{...(2b)}$$

where p is the hydrostatic pressure

$$p = \frac{1}{3}(\sigma_r + 2\sigma_c) \qquad \text{...(2c)}$$

One of the boundary conditions requires that the displacement at the cavity wall, Lagrangian coordinate r=0, is given by

$$u(0,t)=Vt, \qquad V=V_z\tan\theta. \qquad \text{...(3)}$$

where V_z and θ are the axial velocity and half cone angle for the conical penetrator. The other boundary condition requires that the radial displacement at the wave front is zero.

Certain transformations are applied on the above equations which results in a nonlinear ordinary differential equation that is solved numerically by a Runge-Kutta integrating subroutine. The axial stress and the axial force on the penetrator is given by

$$\sigma_r = \alpha V_z \tan\theta (K\rho_0)^{1/2} \quad \text{...(4a)}$$

$$F_z = \alpha \pi r^2 V_z \tan\theta (K\rho_0)^{1/2} \quad \text{...(4b)}$$

where α and r are slope of the linear fit to the stress curves and the radius of the penetrator respectively. If the axial velocity V_z is now permitted to vary with time then from equation (4b) the equation for rigid body motion of the penetrator with mass m is

$$m\frac{dv}{dt} = -bv, \qquad b = \alpha\pi r^2 \tan\theta (K\rho_0)^{1/2} \quad \text{...(5)}$$

Equation (5) with the initial condition $v(t=0)=V_0$ has solutions

$$s = mV_0/b[1 - \exp(-bt/m)] \quad \text{...(6a)}$$

$$v = V_0 \exp(-bt/m) \quad \text{...(6b)}$$

$$a = -bV_0/m \exp(-bt/m) \quad \text{...(6c)}$$

where s, v and a are distance, velocity and acceleration of the penetrator at a particular time t.

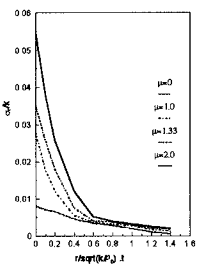

Fig. 4 Radial Stress Profiles

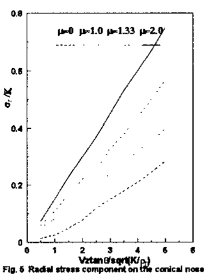

Fig. 5 Radial stress component on the conical nose

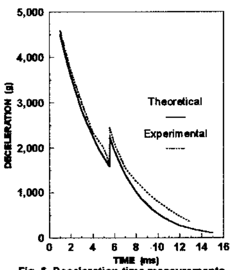

Fig. 6 Deceleration-time measurements

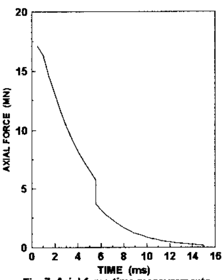

Fig. 7 Axial force-time measurements

Comparison With a Field Test: Using above technique an actual field problem is analyzed. This experiment had been carried out at the Sandia, Tonopah Test Range, Nevada. Deceleration from the experimental and theoretical results are compared in fig. (6). They are in good agreement for practical test range. Slight discrepancy is due to the fact that experiment was carried out with a penetrator having ogive nose profile while proposed model is for conical penetrators.

Conclusion: Broad conclusion drawn by the results is that for target, shear strength while for projectile, velocity are the most important parameters affecting the target response. Radial stress increases with soil parameter μ as well as velocity of projectile as shown in fig. (4) and (5) respectively.

References:

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