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DYNAMIC ANALYSIS OF CABLE-STAYED BRIDGES INCLUDING CABLE MOTION BY GALERKIN METHOD

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1. INTRODUCTIONS: Dynamic analysis of long-span cable-stayed bridges including cable motion is recently of great interest. To include cable motion, Nagai et al [1] applied the displacement field of cables that is obtained by eigenvalue analysis of each cable and is transformed to modal coordinate in advance. Although this method leads to the satisfactory results, the calculation is laborious. In this paper, the simplified method for dynamic analysis of cable-stayed bridges including cable motion is presented by the Galerkin and finite element methods. The numerical examples are performed on the Tatara Cable-Stayed Bridge with the main span of 890 m.

2. FREE-VIBRATION AND STRUCTURAL DAMPING ANALYSES

The matrix equation of motion for a freely vibrating damped system can be written as

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{0} \qquad (1)$$

where **M**, **C** and **K** are, respectively, the structural mass, damping and stiffness matrices; **u** is the displacement vector. The displacement vector can be expressed in terms of a set of assumed shape $\boldsymbol{\Phi}$ of generalized-coordinate amplitude **a** as

$$\mathbf{u} = \boldsymbol{\Phi}\mathbf{a}e^{\omega t} = \sum_{r=1}^{N} \phi_r a_r e^{\omega t} \qquad (2)$$

where ω is the circular frequency, and N is a number of assumed modes. $\boldsymbol{\Phi}$ can be reasonably approximated from the static defection curves of deck-tower and cables (Fig. 1) under various load conditions. These deflection curves include in-plane, out-of-plane, and torsion modes of deck-tower, and first in-plane and out-of-plane modes of cables.

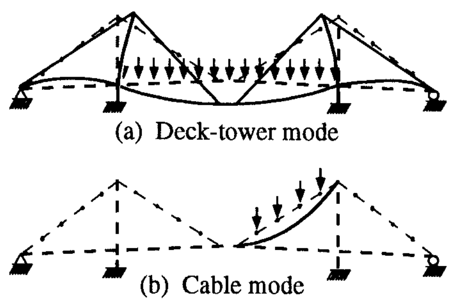

(a) Deck-tower mode
(b) Cable mode

Fig1. Assumed mode shapes from static deflection curves with each modal damping factor

Using Galerkin method, differentiating and substituting Eq. (2) into Eq. (1), and adding another set of equation to Eq. (1), lead to

$$\omega\begin{bmatrix} \mathbf{0} & \mathbf{M}_g \\ \mathbf{M}_g & \mathbf{C}_g \end{bmatrix}\begin{Bmatrix} \omega\mathbf{a} \\ \mathbf{a} \end{Bmatrix}e^{\omega t} + \begin{bmatrix} -\mathbf{M}_g & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_g \end{bmatrix}\begin{Bmatrix} \omega\mathbf{a} \\ \mathbf{a} \end{Bmatrix}e^{\omega t} = \begin{Bmatrix} \mathbf{0} \\ \mathbf{0} \end{Bmatrix} \quad (3); \qquad \left.\begin{matrix} \mathbf{M}_g = \boldsymbol{\Phi}^t\mathbf{M}\boldsymbol{\Phi}, \quad \mathbf{K}_g = \boldsymbol{\Phi}^t\mathbf{K}\boldsymbol{\Phi} = \boldsymbol{\Phi}^t\mathbf{F} \\ \mathbf{C}_g = \boldsymbol{\Phi}^t\mathbf{C}\boldsymbol{\Phi} \cong \mathrm{diag}(2\zeta_r\omega_r M_r) \end{matrix}\right\} (4)\text{-}(6)$$

where $\mathbf{M}_g$, $\mathbf{C}_g$ and $\mathbf{K}_g$ are, respectively, the generalized mass, damping and stiffness matrices; ζ_r is the modal damping factor; and **F** is the load vector. Then, the damped free vibration can be calculated from the complex eigenvalue analysis of

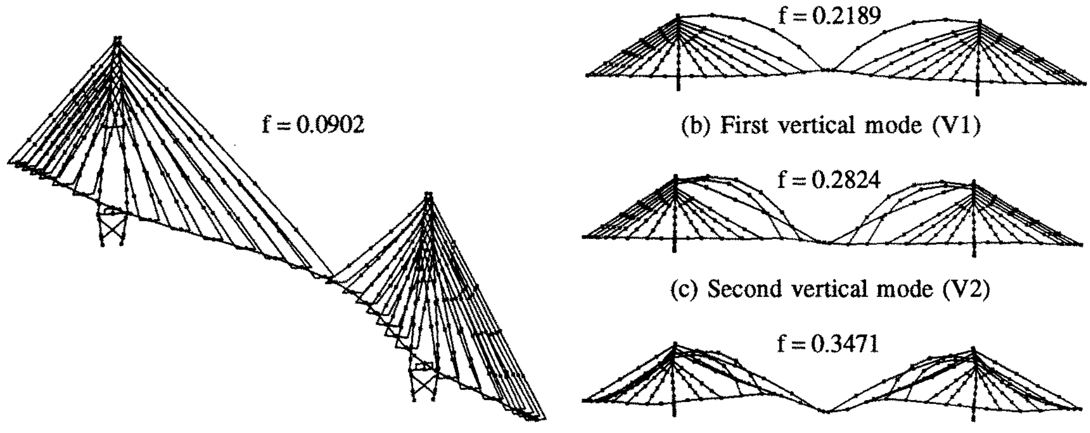
$$\mathbf{D}\mathbf{y} = \lambda\mathbf{y} \quad \text{where} \quad \lambda = \frac{1}{\omega}, \quad \mathbf{D} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{K}_g^{-1}\mathbf{M}_g & -\mathbf{K}_g^{-1}\mathbf{C}_g \end{bmatrix}, \quad \mathbf{y} = \begin{Bmatrix} \omega\mathbf{a} \\ \mathbf{a} \end{Bmatrix} \qquad (7)\text{-}(10)$$

and **I** is the identity matrix. For a stable system, ω will either be real and negative or complex ($\omega = \omega_R + i\omega_I$, $i = \sqrt{-1}$) with a negative real part. Finally, the structural damping can be calculated from

$$\zeta = -\omega_R \Big/ \sqrt{\omega_R^2 + \omega_I^2} \qquad (11);$$ where ω_I is the damped natural frequency.

3. NUMERICAL RESULTS: In the mathematical model of Tatara Bridge, Each cable is modeled by five truss elements with E_{eq}, a deck and towers are idealized by a beam-column element. 35 deflection curves of deck-tower and cable modes are used. Fig. 2 shows the dynamic interaction between cables and a deck-tower system in the V1, V2 and V3 modes, but not in the H1 mode.

For the damped deck-tower vibrations, the logarithmic decrements are assumed to be 0.03 and 0.02 for the flexural and torsional modes, respectively. For the damped cables, 5 sets of the logarithmic decrements are assumed to be 0.001, 0.02, 0.05, 0.1, and 0.3. Fig. 3 shows that the damped natural frequencies of the first few modes do not vary with the increase in cable damping. Fig. 4 shows dependence of structural (system) damping upon the cable damping and structural modes. The structural



(a) First horizontal mode (H1)

(b) First vertical mode (V1)

(c) Second vertical mode (V2)

(d) Third vertical mode (V3)

Fig. 2 Undamped natural mode shapes and frequencies

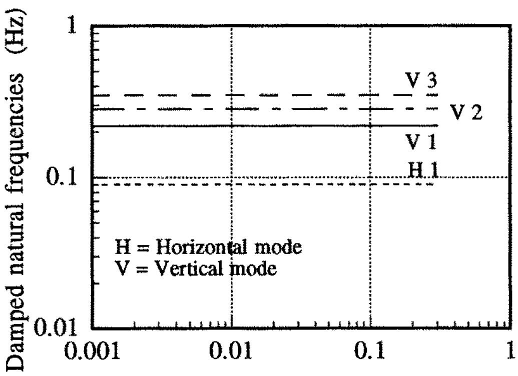


Fig. 3 Damped natural frequencies

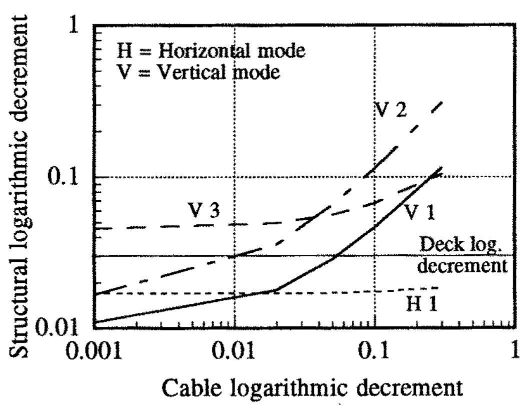


Fig. 4 Dependence of structural damping upon cable damping and structural modes

damping of H1 mode do not increase with increase in cable damping. This is because the stay cables will be acted as a dynamic absorber only when the natural frequency of certain cables is close to the natural frequency of the structure. Fig. 5 shows equivalent mass of deck.

4. CONCLUSIONS: The simplified procedure for dynamic analysis of cable-stayed bridges including cable motion is derived by the Galerkin and finite element methods. The conclusions are: (1) the effects of cable motion are important; (2) increase in structural damping in the vertical modes can be obtained by increase in cable damping; and (3) dependence of structural damping upon structural modes can be found.

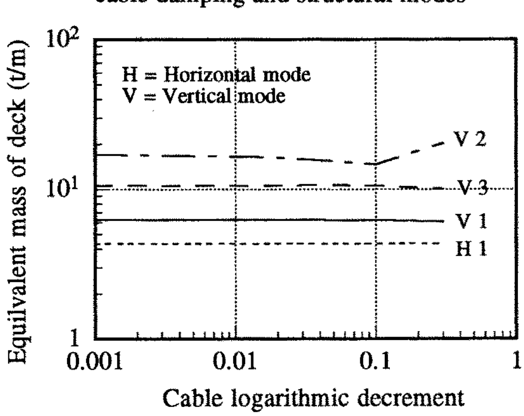


Fig. 5 Equivalent mass of deck

5. REFERENCE: [1] Nagai, M., et al., "Three dimensional dynamic analysis of cable-stayed bridges including cable local vibration", *4th Asia-Pacific Conf. on Struc. Eng. & Const.*, Seoul, Koral, pp. 1845-1850, 1993.