I -452 Application of Robust Control to the Flutter on Long Span Bridges.

Nguyen Ngoc DUNG, Student member, Toshio MIYATA, Hitoshi YAMADA,
members JSCE, Yokohama National University.

Introduction: Recent development of longer span covered by the suspension bridge have made this kind of structure more susceptible to the flutter and as such, raise the concern of this failure in the design process. In fact, experiences and studies have shown that if no special measure is taken, the wind speed of flutter onset could be dangerously low. Increasing the flutter resistance by passive means as reinforcement of the deck, change the deck section, etc. have only yielded some limited results with compromising other designing aspects. Therefore the application of active control seems to be the logical choice. However, due to the nature of the flutter as the system stability, special considerations should be addressed to the problem of uncertainties and stability. In ref.[2] the idea of robust control with a static state feedback was presented. In this study, a more sophisticated control scheme with the dynamic output feedback is explored.

Flutter Problem in View of Control Application: The equation describes the motion of a section of bridge deck as in the fig. 1 can be expressed as:

$$\mathbf{M}.\ddot{\mathbf{r}}+\mathbf{K}.\mathbf{r}=\mathbf{f}_w+\mathbf{w} \quad ; \mathbf{M}=\mathrm{diag}.[mb, I_\alpha]\,,\ \mathbf{K}=\mathrm{diag}.[mk_h, k_\alpha] \qquad (1)$$

With $\mathbf{r}=\{h/b,\alpha\}^T$, $\mathbf{w}$ is the motion-independent disturbances and the $\mathbf{f}_w$ is the aerodynamic forces. For a harmonic motion of frequency ω, according to Scanlan (1993)[3], this later term can be expressed in function of the states and a set of experimental determined coefficients H_i,A_i. These are function of the reduced frequency $k=b\omega/U$ and this relationship can be shown as

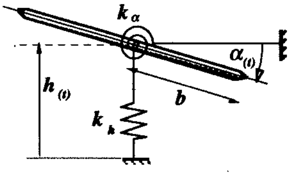

Fig. 1 Model of bridge deck

$$\mathbf{f}_w=\omega\mathbf{F}_V\dot{\mathbf{r}}+\omega^2\mathbf{F}_D\mathbf{r}=-[i\mathbf{F}_V+\mathbf{F}_D].\ddot{\mathbf{r}}=\mathbf{F}_W.\ddot{\mathbf{r}}\ ,\ \mathbf{F}_V=\rho b^3\begin{bmatrix}2H_1^* & 4H_2^*\\ 4bA_1^* & 8bA_2^*\end{bmatrix},\quad \mathbf{F}_D=\rho b^3\begin{bmatrix}2H_4^* & 4H_3^*\\ 4bA_4^* & 8bA_3^*\end{bmatrix} \qquad (2)$$

where the simple relationship between displacement, velocity and acceleration of the harmonic motion is taken into account. In view of (2), the equation (1) now becomes a set of equations parameterized by the reduced frequency k:

$$(\mathbf{M}+\mathbf{F}_w).\ddot{\mathbf{r}}+\mathbf{K}.\mathbf{r}=\mathbf{w} \qquad (3)$$

If a feedback control is applied to the system (3), the whole system in the state form can be reduced to:

$$\dot{\mathbf{x}}=\mathbf{A}.\mathbf{x}+\mathbf{B}_1.\mathbf{w}+\mathbf{B}_2\mathbf{u}\ ,\quad \mathbf{y}=\mathbf{C}_1.\mathbf{x}+\mathbf{D}_{12}.\mathbf{u}\ ,\quad \mathbf{z}=\mathbf{C}_2.\mathbf{x}+\mathbf{D}_{21}.\mathbf{w} \qquad (4)$$

where $\mathbf{x}$ is the state variable, $\mathbf{y}$ is the controlled output, $\mathbf{z}$ is the measured output, $\mathbf{w}$ is the disturbances including noise on sensors and the control in general can be expressed as a dynamic system:

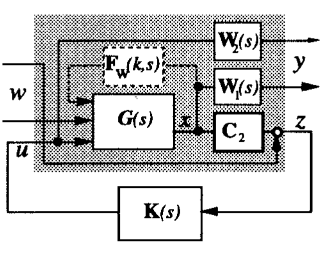

Fig. 2 Control setup.

$$\mathbf{K}_{(s)}:=\{\dot{\mathbf{q}}=\mathbf{A}_c.\mathbf{q}+\mathbf{B}_c.\mathbf{z}\quad ,\quad \mathbf{u}=\mathbf{C}_c.\mathbf{q}+\mathbf{D}_c.\mathbf{z}\} \qquad (5)$$

The design task is the determination of $[\mathbf{A}_C,\mathbf{B}_C,\mathbf{C}_C,\mathbf{D}_C]$ according to certain performances criteria. For the H_∞ control, the "performance" is to reduce the H_∞ norm of the close loop transfer function $\mathbf{G}_K$:

$$\|\mathbf{G}_{K(s)}\|_\infty=\sup_{\omega\in\Re}\rho[\mathbf{G}_{K(s)}(i\omega)]<\gamma \quad \text{and} \quad \sigma(\mathbf{A}+\mathbf{B}.\mathbf{F})\subset\left[C^-:=\{s\in C|\mathrm{Re}\ s<0\}\right] \qquad (6)$$

The open loop of the system included the aerodynamic forces is a varying plant parameterized by k. As the aerodynamic functions are determined by experimental data, they might inherit some degree of uncertainty. The objective is to devise an unified control which gives acceptable stability for all the plants. i.e., a robust control insensible to the varying parameter k.. From the robust control theory, this can be achieved by designing K(s) for a generalized plant basing on a nominal Go and scaling up the outputs with certain frequency dependent weighting functions W1,W2 as shown in fig. 2. The control objective then becomes:

$$\left\|\begin{matrix}\mathbf{W}_2(\mathbf{I}-\mathbf{GK})^{-1}\\ \mathbf{W}_1\mathbf{K}(\mathbf{I}-\mathbf{GK})^{-1}\end{matrix}\right\|_\infty<\gamma \quad \text{with} \quad \rho_{\max}[\mathbf{W}_1]\geq\rho_{\max}[\Delta\mathbf{G}_i] \quad \text{where} \quad \Delta\mathbf{G}_i=\mathbf{G}_i-\mathbf{G}_o \qquad (7)$$

In this case, the nominal plant can be chosen based on $k=k_o$, then the singular values of other plants determined by k_i will be analyzed to decide the weighting functions. Once the generalized plant is formed, the control can be readily derived as in [1] and the close loop system can be analyzed by any known method of flutter prediction.

Numerical Example: To investigate the effect of this control method, a 2D model of the bridge section as shown in fig. 1 is used. The main parameters of model are: $m=2.95\times10^3$ kgf-m/s^2; b=15m; $I_\alpha=5.88\times10^5$ kgf-m^3/s^2; $k_h=447$ kgf.m/m; $k_\alpha=4.68\times10^5$ kg-f/m. The deck is considered as a flat plate and therefore, their aerodynamic coefficients are computed from the Theodosen function. The natural frequencies of this structure is very low, 0.892 rad/s for the pitching and 0.389 rad/s for the heaving motions. The wind speed at which flutter occurs is very low, just over 50m/s. The control will be based only on the twisting moment, which can be generated by a rotating cylinder or by an eccentric weight as suggested in [2]. Firstly, appropriate weighting functions should be designed. For that purpose, the singular values of transfer function G_i are plotted against k and ω (fig. 3). It can be observed that the structure could become more unstable near the origin and small k. However, as the whole plane is divided by the constant U lines, not all of these points have to be considered. Specifically, all points below the maximum design wind speed line can be considered as unreal and discarded. This may suggest that the range of design wind speed could be prefixed beforehand and the weighting functions are determined accordingly. For this example, a Umax=100 m/s is assumed for the plot in fig. 4. Here W_1 is chosen to penalized the observed output in the low bandwidth where the structure modes are expected. By contrast, the function W_2 is applied to the controller output therefore, it penalizes on the high frequency responses of actuator. Such a strategy can make the controller energy more concentrated on the interested frequency range and at the same time, reduce the sensitivity where the noise could be disturbing.

Once the weighting functions are resolved, a generalized plant is build as shown in the fig. 2. It should be noted that there are still many choices on the selection of the outputs to regulated among the state of the system which give rise to different set of control. In this example, the control was devised with the heaving motions are mainly targeted . The implementation of this control as a compensator is applied to the system of equation and the model is subjected to a flutter analysis with the results are shown in the fig. 5 together with the uncontrolled responses. It could be observed that not only the flutter have been effectively eliminated but the control has increased the overall damping of the system. However, as K is a dynamic system, the number of states is increased and more modes have been added to the final system.

Concluding Remarks: The results of this study has convincingly shown that robust control methods can be effectively applied to improved the flutter resistance of the bridge deck. It also provides a simple and convenient way to tangle the problem of uncertainties which has always been a great concern in the application of active control to the civil engineering structures in general.

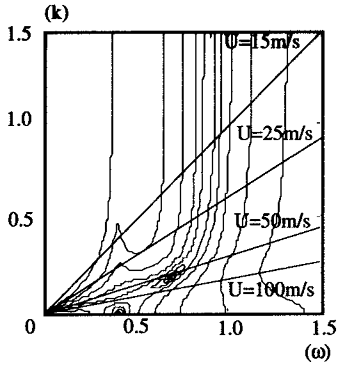

Fig 3 Contour plot of singular value in ω-k plane.

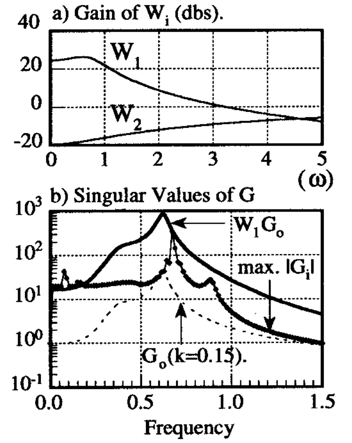

Fig. 4 The weighting functions a) and the singular values plots of open loop b).

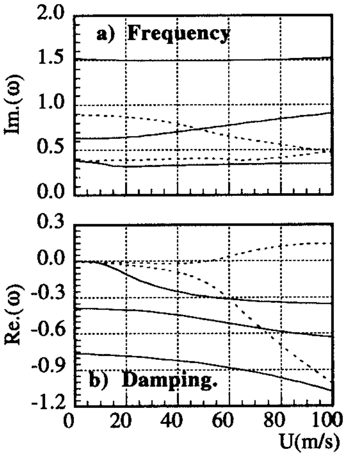

Fig. 5 Flutter analysis of the model, dash line is the uncontrolled responses.

References:

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